# Fundamental Physics Notes

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**QRECT VIDEO VERSION 2015**

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# Preface

These notes have been compiled in order to summarize the core concepts, definitions, terms, equations, and relationships for an introductory college level Physics course. My objective is to provide the student with the notes which serve as a guide to my lectures and an outline of the course. There are a large number of very well written texts that are available. But it is easy for a student to become overwhelmed in the more than one thousand page texts. Thus these notes are the skeletal framework upon which one can attach the rest of the material where a chapter is reduced to less than a single page.

I have separated each 'chapter' into a separate sections or modules that are small but cohesive concepts. I have posted these notes on the web thus allowing one to print these pages for personal use. Each of these sections or modules is designed to support a videotaped segment which are available by web. Each module or video segment can be followed by questions, or problems, to which the student is to respond in the QRECT learning assessment system. These questions are to be part of the student's daily grade and to guide both the student and the instructor in the assessment process. This design insures a higher level of engagement by the student and is designed to simulate one-on-one instruction (tutoring) for any number of simultaneous students.

The lecture sections can be presented in a synchronous class interspersed with the student responses submitted simultaneously in real time by all students (using internet connected devices such as iPhones, iTouches, iPads, netbooks or any internet device) into the QRECT software server. The lectures can be augmented by instructor comments, partial lectures, class demonstrations, or problem solving explanations. The material can also be offered in a synchronous distance education environment or even in self-directed individual asynchronous environments. As a self-directed or 'self-paced' course, it is possible to reroute the student if performance is not adequate to proceed. It is also possible for students to achieve a very high performance rate for domains where they are more capable. The advantages of videotaped lectures are (a) the instructor can replicate themselves and achieve much higher lecturing efficiency. (b) The student can review material many times as may be useful. (c) There are less time restrictions on the student thus providing the material that was missed due to illness or other causes such as athletic events. It also allows course scheduling flexibility. (d) The instructor can augment the core lectures with additional lectures, demonstrations, problem solving sessions all of which can also be videotaped thus extensively enriching the information available to the student. (e) The system also provides the infrastructure for a fully self-paced course. I have used red fonts for equations and green fonts for numerical values and constants thus providing a rapid recognition. I have developed web based software for UNITS conversion that allows one to mix units in any valid way thus providing an environment for very rapid computation. The general Class Notes, Video Lectures, UNITS software, and the QRECT software all can be found at www.asg.sc.edu. I welcome comments and suggestions (at jjohnson@sc.edu).

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**Mechanics**

# 1 Measurement & Vectors

## 1.1 Data, Units, & Metadata (video)

- For thousands of years, scientists have used mathematics to represent scientific information
  - ♦ This consists generally of a numerical value, units, and the descriptive metadata
    - ◊ For example: <4.6 | Kilograms | Brown Rice >
  - ♦ The 'number' might be a real or could be complex number, a vector, or a tensor array
- Early English Units (using the correct UNITS names):
    - ◊ Length: inch, hand, foot, cubit, yard, fathom, mile,
    - ◊ Area: square foot (ft2), acre, square mile
    - ◊ Volume: fluid ounce (ouncef), pint, quart, gallon, barrel, cubic foot (ft3),
    - ◊ Time: second (s), minute (min), hour, day, fortnight, month, year, century
    - ◊ Mass: pound, ton, stone
- Try UNITS: = 4*yard/inch, = 100*year = 16* gallon/ouncef

## 1.2 SI (Scientific International Units – Napoleon about 1800) – Primary Units Space, Time, Mass (video)

- ♦ Length: Meter = distance that light travels in a vacuum in 1/299,792,458 s (since 1983)
  - ◊ Originally 10-7 of the distance from the equator to the north pole. (1799)
  - ◊ Until 1960, the distance between two lines on a platinum iridium bar in Paris
  - ◊ In 1960 was defined as the 1,650,763.73 wavelengths of Krypton 86 light
  - ◊ Scales of distance: quark-quark, atom, virus, human, earth, to sun, universe
- ♦ Mass: Kilogram = the mass of a platinum-iridium cylinder in Paris
  - * (mass of 1/1000 of $m^3$ of water)
  - ◊ Scales of masses: electron, proton, .. human, planet, star, galaxy
- ♦ Time: Second = the time of 9,192,631,770 vibrations of Cesium 133 radiation
  - ◊ Before 1960 was 1/86,400 of avg. solar day (60 s / min, 60 min/hr, 24 hr /day)
  - ◊ Scales of time; light across proton, cesium, human lifetime, age of earth, universe
- ♦ Electrical Current: Ampere = the current flowing in both of two parallel infinite wires that results in a force of 2E-7 Newtons / m
  - ◊ The Coulomb is defined as Coulomb = Ampere * Second
- ♦ Temperature: Kelvin = 273.16 K is defined as the temperature above absolute zero for the triple point of water-ice-steam in equilibrium (at a temperature of 0.1C and a water vapor pressure of 610 Pa

## 1.3 Unit Conversions (video)

- ♦ +- only of same types, */ any kinds, transcendental functions (dimensionless)
- ♦ Derived units: m/s, kg/m, $m^2$ $m^3$
- ♦ Unit conversion is achieved by forming unity with which one can multiply any expression

## 1.4 Prefixes, Powers, Greek Alphabet as Symbols (video)

- ♦ Powers of 10 & Prefixes
  - ◊ Kilo $10^3$, Mega $10^6$,Giga $10^9$, Tera $10^{12}$, Peta $10^{15}$, Exa $10^{18}$ , Zetta $10^{21}$, Yotta $10^{24}$
  - ◊ Milli $10^{-3}$, Micro $10^{-6}$ Nano $10^{-9}$ Pico $10^{-12}$ Femto $10^{-15}$ , Atto $10^{-18}$, Zepto $10^{-21}$,Yocto $10^{-24}$
  - ◊ Hecto $10^2$, Deka $10^1$ , Deci $10^{-1}$ , Centi $10^{-2}$,
- ♦ Common names: dozen, gross, ream, thousand, billion, trillion, quadrillion,…
- ♦ Use of the Greek Alphabet as additional symbols
  - ◊ αβγδεζηθικλμνξοπρστυφχψω ΑΒΓΔΕΖΗΘΙΚΛΜΝΞΟΠΡΣΤΥΦΧΨΩ

## 1.5 Scientific Notation & Numerical Uncertainty (video)

- ♦ 123.4 = 1.234E2 = 1.234 x $10^2$ (always lead with 1 digit then decimal)
- ♦ Numerical Uncertainty
  - ◊ Rules for addition and multiplication with numerical uncertainty

# 2 Kinematics in One Dimension

## 2.1 Motion of an object in space - Define velocity & acceleration (video)

- The fundamental concepts of motion.
  - A single mass moves in three dimensions of space in time
  - Motion in three dimensions can be viewed as three independent one dim. motions
  - The internal behavior of the single mass can be ignored.
    - Treat its position is at the center of mass
  - The 'state of a particle' is given by the position and velocity at one instant of time
  - We seek to predict the future motion of a mass: given position and velocity at one time
    - i.e. predict its motion: given x(0) and v(0) then what is x(t) and v(t)
- Define velocity and acceleration (average and instantaneous)
  - Define average velocity (instantaneous velocity $v = dx(t) / dt$ )
  - Define average acceleration (instantaneous acceleration $a = dv(t) / dt$
  - Examples (falling mass; mass thrown upward)

## 2.2 Motion of one particle in one dimension: (video)

- Constant acceleration & constant velocity equations
  - When velocity is constant $v(t) = v(0)$ and $x(t) = x(0) + v(0)\, t$ and thus $a(t) = 0$
  - When acceleration is constant and $x(t) = x(0) + v(0)\, t + \frac{1}{2} a\, t^2$
  - Another equation is obtained on eliminating time: $v(t)^2 - v(0)^2 = 2\, a\, d$ where $d = x(t) - x(0)$
  - Proof: begin with $d^2x/dt^2 = a$
  - Examples

## 2.3 Constant gravity problems (video)

  - On Earth: $a = g = 9.8\ m/s^2$ or $= 32\ f/s^2$
  - $v(t) = 0$ at top of motion
  - $a(t) = a = g$ all the time
  - $v(0) = v(t)$ when the object again falls to the same height
  - Problems:
  - Terminal velocity – of a human 140 mi/hr max drag (spread);
    - 240 mi/hr min drag (standing)

# 3 Kinematics in Two & Three Dimensions

## 3.1 Vectors and Vector Products (video)

- Vectors Addition, Subtraction, & multiplication by a constant
  - This forms what is called a Linear Vector Space
  - The dimension of a space is the number of numbers needed to specify a point.
  - Graphical method described (only works in two dim. but is useful for visualization)
  - ijk method – Do not use – this is antiquated and awkward
  - Component form: $(x, y, z) = (x^1, x^2, x^3) = x^i$
  - $(A_1,A_2,A_3) + (B_1,B_2,B_3) = (A_1 + B_1, A_2, + B_2, A_3 + B_3)$ and $a^* (A_1,A_2,A_3) = (aA_1, aA_2, aA_3)$
- Vector Product**s**
  - Scalar Product: A linear vector space with a scalar product is called a ‘Metric Space’
    - ◊ $\mathbf{A} * \mathbf{B} = \mathbf{A} \bullet \mathrm{B} = A_xB_x + A_yB_y + A_zB_z = AB\cos\theta$ (a scalar value)
    - ◊ The dot (scalar) product works in any number of dimensions
    - ◊ The dot product contains the Pythagorean theorem!
    - ◊ Even more generally a curved (Riemannian) geometry
      - * $A^*B = g_{\mu\nu} A^\mu B^\nu$ where $g_{\mu\nu}$ can be a function of the coordinates
  - Cross Product $(\mathbf{A} \times \mathbf{B})_i = \varepsilon_{ijk} A_j B_k = AB\sin\theta$ in magnitude with direction from RHR
    - ◊ Used with especially with torque and magnetic forces

## 3.2 Projectile motion in two dimensions using vectors r(t) = (x(t) , y(t) ) and v(t) = ($v_x$(t) , $v_y$(t)) (video)

- Vertical motion is like one dimensional motion with constant a = g
- Horizontal motion is as though a =0 and thus v = constant
- Combined motion of vertical & horizontal
  - Compare to view of one dimensional motion from a moving car or train
- Problems (projectile motion)

## 3.3 Graphical view of motion in a river or with an air current using vectors graphically (video)

- Compound the motion by adding vectors of person relative to water and water to ground.
- Determine angle of real motion, angle necessary to stay still, time across water etc
- Similar problem of combined velocity of airplane & wind velocity

## 3.4 More complex projectile problems (video)

- Projectile which goes over a cliff
- Projectile in moving air

# 4 Forces & Newton's Laws of Motion

## 4.1 The concept of a force & Newton's laws (video)

- We intuitively know what a force is – and that it is a vector (has a direction)
  - Mass as a measure of inertia, the resistance to acceleration. - units of kg
  - Inertial reference frame: F = 0 means constant motion (velocity)
- Newton's Laws
  - First Law: **F**=0 implies **a** =0 and conversely
  - Second Law: **F**= m**a** (for constant mass situations) (The second law contains the first)
    - ◊ Force measured in Newton's = Nt = kg m/s$^2$
    - ◊ **The exactly correct equation is:** **F**= d**p**/dt or = Δ**p**/Δt where **p** = m **v**
    - ◊ Example & problem
  - Third law $\mathbf{F}_{1\text{->}2} = -\mathbf{F}_{2\text{->}1}$
    - ◊ Newton also erroneously stated that the forces were along the lines of centers
    - ◊ He did not know about magnetic forces

## 4.2 The Fundamental Forces (video)

- Gravitational Force $F_{grav} = G\, m_1\, m_2 / r^2 = m\,(G\,M / R^2) = m\,g$ (relative strength of $10^{-39}$ )
  - and Near the earth's surface $F_{grav}$ = Weight = W = mg
  - Thus $g = GM/R^2$ which can be used to give g on other planets.
  - (and affects all masses and even pure energy (light) – infinite range)
- Weak force (relative strength of about $10^{-14}$ )
  - involves leptons and neutrinos, very short range)
- Electrical & Magnetic Force $F_{em} = q\,E + q\,v \times B$ where $F = k\, q_1\, q_2 / r^2$
  - (involves charged particles and currents – infinite range – strength of $10^{-2}$)
  - Note how similar the form is to the force of gravity (but there is no negative mass)
- Strong (nuclear force and between quarks about 1' or '10')
  - range of $10^{-15}$ m: p & n bound by pions) 1 / Strong (quarks bound by gluons)

## 4.3 Derivative Forces: (video)

- Frictional Force (static & dynamic) $F_{fric} = \mu\, F_{normal}$
- Elastic Force near equilibrium $F_{elas} = -kx$ where x is the distance from equilibrium
  - ◊ ( Hook's law)
- Centripetal force $F_{cen} = m\, v^2 / r$ where r is the radius of curvature
- Force of tension is equal to the force with which the rope is pulling.
  - ◊ Equilibrium as $\mathbf{F}_{total} = 0$

## 4.4 Resolution of forces & their vector nature (video)

- Attwood's Machine
  - Force of tension
- Incline plane
  - Without friction – one mass
  - With friction – one mass
  - Force of tension

## 4.5 More difficult problems with forces (video)

- With friction and two masses - tension
- Problems with vector force resolution
  - Problem with rope stretched horizontally with weight

# 5 Uniform Circular Motion

## 5.1 Circular motion and centripetal acceleration and force (video)

- Definition of uniform circular motion with velocity v and radius r
- Centripetal (means moving toward a center) acceleration
- Period T of circular motion is defined by $v = 2\pi r / T$
    - That is one circumference in one period
- $a_{cen} = v^2 / r$ thus $F_{cen} = m\, a_{cen}$

## 5.2 Problems and examples I (video)

- Problem of balancing friction with centripetal forces of a car driving around a curve– flat road
- Same problem of car on a curve but with a road that is angled

## 5.3 Problems and examples II (video)

- Problem of satellites in circular orbit $GmM/r^2 = m\, v^2/r$ thus $v = (GM/r)^{1/2}$
- Artificial gravity using circular motion
- Problem of pail of water rotated in a vertical plane

# 6 Work & Energy

## 6.1 Concepts of work and energy (video)

- Work requires energy and they are often considered synonymous –
  - ♦ Energy is conveyed from one system to another exactly by the work done.
  - ♦ More precisely, an increase in energy is always equal to (and due to) work that is done
  - ♦ Work and energy are scalar quantities with no direction since they are direct products of vectors ($dW = \mathbf{F} \bullet \mathbf{dr} = \mathbf{F\ dr\ cos(\theta)}$ ).
- Types of energy:
  - ♦ Kinetic – energy of motion
  - ♦ Potential – energy due to position or configuration such as
    - ◊ Gravitational potential (higher has more energy)
    - ◊ Spring (elastic) potential (more compressed or stretched has more energy)
  - ♦ Chemical – stored in potential chemical reactions of atoms and molecules
    - ◊ usually where carbon is "burned" or combined with oxygen.
    - ◊ Food energy as measured in Calories (capital 'C' means Kilocalories)
  - ♦ Nuclear – stored in potential nuclear reactions that can release heat energy
  - ♦ Solar & radiant – energy from light and more generally electromagnetic radiation
  - ♦ Heat – energy due to the random motion of molecules and constituents

## 6.2 Definition of Work & Energy (video)

- Work = $dW = \mathbf{F} \bullet \Delta\mathbf{r} = F\,\Delta r \cos\theta$ with units: Joules = Newtons * meters or J = N m
  - ♦ More exactly, using calculus: $W = \int \mathbf{F} \bullet \mathbf{dr}$
  - ♦ The unit of work is the Joule (J) = 1 Nt acting through 1 m i.e. 1J = 1Nt*1m
  - ♦ If the motion is along a straight line & the force is at a constant angle, then W = F x cos(θ)
  - ♦ The force is conservative if this integral is path independent (or zero for any closed curve) Study **c**onservative and nonconservative forces – path independence of work reversibility

~6.2:1 If an object if moved a distance of {4.5*m} along the X axis by the action of a force of {7.2*n} in the direction of motion, in a time of {4* s} how much work is done?.

~6.2:2 If an object if moved a distance of {4.5*m} along the X axis by the action of a force of {7.2*n} acting at an angle of {25*deg} to the direction of motion, in a time of {4* s} how much work is done?.

- Kinetic Energy: $KE = \frac{1}{2} mv^2$
  - ♦ Kinetic Energy $KE = dW = \mathbf{F} \bullet \mathbf{dr} = \mathbf{m\ (dv/dt)\ dr} = \mathbf{m\ v\ dv}$ thus $KE = \frac{1}{2} mv^2$

~6.2:3 If an object at rest is pushed with a force of 1.3*n

- Gravitational Potential Energy $PE = mgh$
  - ♦ Gravitational Potential Energy $dW = \mathbf{F}_{grav} \bullet \mathbf{dr} = m\,g\,dh$ or $PE = mgh$
- Elastic Potential Energy $PE = \frac{1}{2} kx^2$
  - ♦ Elastic Potential Energy $dW = \mathbf{F}_{elas} \bullet \mathbf{dr} = kx\,dx$ thus $PE = \frac{1}{2} kx^2$

## 6.3 Power is defined as the rate of doing work or expending energy (video)

- Power = P = Work / time = W / t with units: Watts = Joules / sec or W = J / s
  - ♦ Using calculus we define Power exactly as: $P = dW / dt$
  - ♦ Energy is often defined in terms of power times time
    - ◊ $dW = P\,dt$ e.g. KWHR = 1000 J/s *3600 s

## 6.4 Conservation of Energy in a Closed System – Example of Kinetic and Potential of a particle (video)

- In a gravitational field KE + PE = const
  - ♦ Falling object
  - ♦ Incline plane and roller coaster
- With a spring KE + PE = const

# 7 Momentum and Impulse

## 7.1 Momentum (video)

- Momentum $\mathbf{p} = m\,\mathbf{v}$ is a vector and very fundamental as a physics concept
  - There is no special name for the momentum units of kg*m/s
    - For any system of particles with momentum one has
    - $d\mathbf{P}/dt = d(\Sigma\, \mathbf{p}_i)/dt = \Sigma_{ji}\, \mathbf{F}_{j\ on\ i} + \Sigma_i\, \mathbf{F}^{ext}_i = 0 + \mathbf{F}^{ext}_{total}$ because $\mathbf{F}_{j\ on\ i} = -\,\mathbf{F}_{i\ on\ j}$
    - Thus if there is no total external force on a system, the internal forces cancel
    - and thus the total internal momentum is conserved.
- Conservation of momentum in a closed system
  - Thus momentum is conserved if there are no external forces

## 7.2 Totally inelastic collisions (momentum is conserved but energy is not) (video)

- Objects stick together after collision & the maximum possible loss of KE to heat
  - When objects stick together there is only one v after collision
    - This is solved by conservation of momentum.
  - Ballistic pendulum (bullet into a block of wood – velocity is obtained by height)
  - Example: Two football players where one tackles the other or an auto crash

## 7.3 Elastic (energy is conserved) and Partially Elastic Collisions (some energy is conserved) (video part 1, part 2)

- Elastic collisions: Kinetic energy after collision is same as before collision
  - Problem: 1 dimension – must use cons. of both energy & momentum to compute $v_1$ & $v_2$ after
  - Super ball bounce is essentially to equal to the previous height (elastic & one dimension)
- Partially Inelastic collisions: Some kinetic energy is lost to heat of the objects colliding
  - Example of a bouncing ball – loss of KE is exactly measured by mgh loss in height

## 7.4 Center of Mass, Equations of a system of particles, and Total momentum of a system (video part 1, part 2)

- Define the Center of Mass $\mathbf{R} = \Sigma_i\, m_i\, \mathbf{r}_i / M$ where $M = \Sigma_i\, m_i$ = total mass of the system
  - Recall from above that $d\mathbf{P}/dt = d(\Sigma\, \mathbf{p}_i)/dt = \Sigma_{ji}\, \mathbf{F}_{j\ on\ i} + \Sigma_i\, \mathbf{F}^{ext}_i = 0 + \mathbf{F}^{ext}_{total}$
  - Thus $d\mathbf{P}/dt = d(\Sigma m_i d\mathbf{r}_i/dt)/dt = \mathbf{F}^{ext}_{total} = d(M\mathbf{V})/dt$ where $\mathbf{V} = d\mathbf{R}/dt$ =velocity of COM
  - It also follows that $\mathbf{P} = M\,\mathbf{V}$

## 7.5 Impulse – When the force is very complex: (video part 1, part 2)

- Impulse is defined as the change in momentum of an object such as a baseball when hit
- Thus Impulse is a vector quantity and is often useful when the force is complicated in time
- Momentum is conserved in a system that has no outside forces acting upon it.
- Impulse = $\Delta\mathbf{p} = \langle F \rangle\, \Delta t$ = the average force times the time interval.
  - Problem of hit baseball, & of rain verses hail on car roof (twice the impulse due to recoil)

# 8 Rotational Kinematics

## 8.1 Angular position, velocity, and acceleration for circular motion (video)

- ♦ Circular motion restricts the distance to be a constant r from a given point
- Angular position
- Definition of angle in radians $\theta = s / r$ where s is the arc length subtended & r is the radius
  - ♦ Thus $\theta_{cycle} = 2\,\pi\, r / r = 2\,\pi$ radians = 360 degrees for the arc of an entire circle.
- Define angular velocity $\omega = \Delta\theta / \Delta t$ in units of radians per second or rad/s
  - ♦ Using calculus angular velocity $\omega = d\theta / dt$
  - ♦ Examples
- Define angular acceleration $\alpha = \Delta\omega / \Delta t$ in units of radians per second squared or rad/s$^2$
  - ♦ Using calculus angular acceleration $\alpha = d\omega / dt$
  - ♦ Examples

## 8.2 Connection with translational motion (video)

- ♦ Since $s = r\theta$, it follows that $ds/dt = v_{tan} = r\,\omega$ and $dv/dt = a_{tan} = r\,\alpha$
- If $\alpha$ is constant then it follows that $\omega = \omega_0 + \alpha\, t$ in analogy with $v = v_0 + a\,t$ for translations
- Likewise it follows that $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha\, t^2$ in analogy with $x = x_0 + v_0 t + \frac{1}{2} a\, t^2$
- Combining these equations by eliminating t we obtain $\omega^2 - \omega_0^2 = 2\alpha\theta$

## 8.3 Centripetal acceleration: (video)

- Centripetal acceleration $a_{cen} = v^2/r = r\,\omega^2$
- Rolling motion problems: the tangential velocity is equal to the velocity of the center of the circle
  - ♦ Example

# 9 Rotational Dynamics

## 9.1 Introduction to Angular Force = Torque (video)

- Forces give acceleration in translational motion, torques give angular acceleration in rotation
  - Thus Torque is to rotations as force is to translations
- For solid objects and systems, we can generally model the motion in translation & rotation
- The translation is of the center of mass while the rotation is about the center of mass
- Translational equilibrium has a net force of zero, rotational equilibrium means no torqu**e**
  - Equilibrium problems are solved by requiring that the total torque (and force) are zero
- Torque defined
  - Imagine a system with one fixed point (the axis) and a force is applied a distance **r** away
  - Torque $\tau$ = the distance from the axis to the force application point times the normal force, F sin $\theta$
  - Thus torque is defined as $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ with the right hand rule governing the direction of $\tau$
  - Units of torque are Newtons x meters = Nm
  - Equilibrium is defined by $\Sigma\tau_i = 0$ and $\Sigma\mathbf{F}_i = 0$

## 9.2 Examples of computing torque (video)

  - Problem: Opening a door
  - Problem: Using a lug wrench or screw driver
  - Problem: Force to support the end of a bridge – sum of several torques
- Center of Gravity = Center of mass with weights replacing masses after multiplication by g – prove:
  - How to find the center of gravity of an object - hang it from two points (intersection of verticals)

## 9.3 Moment of Inertia = the angular concept of inertia (mass) or resistance to angular acceleration (video)

- Moment of Inertia defined by $I = \Sigma_i m_i r_i^2$ with units of kg $m^2$
  - $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = r F_{nor} = r\, ma$ (but $a = r\alpha$) thus $\tau = m r^2 \alpha$ which holds for each particle in a system
  - Thus for an ensemble of particles $\tau = (\Sigma_i m_i r_i^2)\alpha = I \alpha$
  - Problem: Moment of inertia for different objects
    - ◊ Solid Sphere $I = 2/5\ MR^2$ ; Hollow Sphere $I = 2/3\ MR^2$ ; Solid Cylinder $I = 1/2\ MR^2$
    - ◊ Rod with axis perp to center $I = 1/12\ ML^2$ ; Rod with axis perp to end $I = 1/3\ ML^2$
  - Problem: Object rolling down a hill

## 9.4 Rotational Kinetic Energy (video part 1, part 2)

  - Rotational Work (Energy) $W = \int \mathbf{F} * d\mathbf{s} = \int (F_{nor}\, r)\ \theta = \int \tau\, d\theta$ thus $W = \tau\, \theta$
  - Rotational Kinetic Energy KE = ½ m $^{v2}$ = ½ m $^{v2}$ = ½ m $^{r2\ -2}$ thus KE = ½ I $^{-2}$
  - Problem: energy of rotating object
- Problem: total kinetic energy $KE = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

## 9.5 Angular Momentum (video)

- Angular momentum: $\tau = r F_{nor} = r\, \Delta p/\Delta t = r\, m\Delta v/\Delta t = r\, mr\, \Delta\omega/\Delta t = \Delta(I\omega)/\Delta t$
- Define angular momentum = $L = I\omega$ then $\boldsymbol{\tau} = \Delta\mathbf{L}/\Delta t$ and compare to $\mathbf{F} = \Delta\mathbf{p}/\Delta t$
  - Using calculus: angular momentum: $\tau = r F_{nor} = r\, dp/dt = r\, m dv/dt = r\, mr\, d\omega/dt = d\,(I\omega)/dt$
  - Define angular momentum = $L = I\omega$ then $\boldsymbol{\tau} = d\mathbf{L}/dt$ and compare to $\mathbf{F} = d\mathbf{p}/dt$
  - Thus for an ensemble of particles $\tau = (\Sigma_i m_i r_i^2)\alpha$ thus $\tau = I \alpha$ like F=ma

# 10 Gravitation

## 10.1 Gravitational Force (video)

- Newton's law of gravitation: Every mass attracts every other mass with a force along lines of centers.
  - With a force:: $\mathbf{F}_{1\to2} = -G\, m_1\, m_2 / |\mathbf{r}_2-\mathbf{r}_1|^2$ directed as an attraction along lines of centers
- Cavendish (1731-1810) was the first to measure the constant $G = 6.673E{-}11\ Nm^2/kg^2$
- Gravity near the surface of a planet:
  - F = m g where for earth g = 9.8m/s = 32 ft/s (approx values)
  - Thus $F_{1\to2} = G\, m_1\, m_2 / |r_1-r_2|^2 = m\,(GM/R^2) = mg$
    - ◊ Where M is the mass and R is the radius of the earth
  - Thus $g = GM/R^2$ is the acceleration due to gravity.

## 10.2 Gravitational Field and vector form of the gravitational force: (video)

- The gravitational field is defined as the force on a unit mass: $F/m = g = GM/R^2$
  - Thus the acceleration due to gravity is also the gravitational field.
- Gravitational Force
  - Newtons law of gravitation on m located at **r**: $\mathbf{F}_{i\to m} = G\, m\, \Sigma_i m_i\, (\mathbf{r}_i-\mathbf{r}) / |\mathbf{r}_i-\mathbf{r}|^3$
  - Gravitational Field: : $\mathbf{g}(\mathbf{r}) = G\, \Sigma_i\, m_i\, (\mathbf{r}_i-\mathbf{r}) / |\mathbf{r}_i-\mathbf{r}|^3$ in units of acceleration $m/s^2$

## 10.3 Einstein's Theory of General Relativity which is a theory of gravity (video)

- In 1916 Einstein's general theory of gravitation showed that even pure energy (eg light) is attracted to a mass or other pure energy.
  - Furthermore gravity was shown to be a curvature of space and time that altered the motion of the mass
  - With black holes, this curvature is so severe that not even light can escape the attraction
- Evidence that the space and time is curved with acceleration – rotating platform
- Evidence that, from symmetry, light should bend in gravity – the elevator thought experiment

# 11 Elasticity

## 11.1 Elastic Distortion of Systems (video)

- When systems are distorted from equilibrium, the restoring force is proportional to the deformation
- Hookes Law: $F = -k\,x$ where a force F causes a proportional deformation x from equilibrium
  - The constant k is called the 'spring constant'
  - The potential energy stored in a deformed system is $PE_{deformaiton} = ½\,k\,x^2$ (=work to deform)
- Taylor series expansion:
  - The Taylor series expansion of the potential is $V(x) = V(0)+dV/dx|_{x=0}\,x+1/2\,d^2V/dx^2|_{x=0}\,x^2$
  - For a particle near equilibrium (x=0) has no force ($dV/dx|_{x=}$ =0) and we can set V(0)=0 as this is an arbitrary constant and does not affect the force F.
  - Thus $V(x) = ½\,k\,x^2$ in lowest order approximation thus giving F = -kx

## 11.2 Other Stress Strain Relationships: (video)

- Generally: Stress is proportional to strain within the elastic limit:
- Young's Modulus:
  - Young's Modulus: Stretch & Compression of solid: F/A = Stress & $\Delta L/L_0$ is the strain
  - $F = Y\,A\,(\Delta L/L_0)$ where Y is the Young's modulus for that substance
    - ◊ and where A is the area where the force F is applied, and $L_0$ is the original length
  - Examples Y values are Brass: 9.0E10, Brick 1.4E10, Steel 2.0E11, Aluminum 6.9E10
  - Note that in some substances, Y for tension (pulling) is different from Y for compression
- Shear modulus:
  - Shear modulus: Forces which create a shear of solid: F/A = Stress & $\Delta X/L_0$ is the strain
  - $F = S\,A\,(\Delta X/L_0)$ where S is the shear modulus for that substance, F is applied force
  - A is the surface area, $\Delta X$ the length of the shear, & $L_0$ is the length of the applied shear
  - Examples S values are: Brass 3.5E10, Steel 8.1E10, Aluminum 2.4E10
- Bulk modulus:
  - Bulk modulus: Pressure on solids, liquids or gasses:
  - P=F/A = Stress & $\Delta V/V_0$ is the strain
  - $\Delta P = -B\,(\Delta V/V_0)$ where Pressure P = F / A in units of $N/m^2$ and B is the Bulk modulus
    - ◊ and $\Delta V$ is the change in volume while $V_0$ is the original volume
  - Examples B values are: Brass 6.7E10, Steel 1.4E11, Water 2.2E9, Ethanol 8.9E8
- Note that units for Y, S, and B are all in Pascal or $N/m^2$

# 12 Simple Harmonic Motion

## 12.1 Harmonic Motion resulting from a system displaced from equilibrium (video)

- Systems distorted from equilibrium and released (without friction), will oscillate about that equilibrium
  - This oscillation has a mathematical form of a sin or cos function, called simple harmonic motion
- Let a mass m, feel a spring force F= -kx where x is the distance from equilibrium. Then:
  - $ma(t)=md^2x/dt^2 = -kx(t)$ has the solution $x(t) = A\cos(\omega t + \delta)$ where $\omega$ = angular velocity
  - For this x(t) to be the solution, one can show that $\omega = \sqrt{k/m}$ must hold
  - A is the amplitude of the oscillation since cos has a range from -1 to +1. It can assume any value
  - The phase of the oscillation = $\delta$ which can assume any value and is determined by x(t=0)
  - A complete cycle occurs by definition in time T, since cos has a cycle of $2\pi$, then $\omega T = 2\pi$
  - Thus the period $T = 2\pi/\omega$. This equation is important since it relates T (intuitive) and $\omega$
  - Since the frequency f is the reciprocal of the period, $f = 1/T$ , then $f = \omega/2\pi$
- The importance of these results are that they describe ANY system near equilibrium (with no friction)

## 12.2 Derivation of Simple Harmonic Motion with Fiction (video)

- Simple harmonic motion (motion of a mass m near equilibrium): $ma(t) = -kx(t) - bv + F_{ext}$
  - Written as a differential equation we get: $m\, d^2x/dt^2 + b\, dx/dt + kx = F_{ext}$ where x = x(t)
  - This is one of the most important equations in physics
  - It also is the equation that describes the RCLV circuit
  - It is a second order (second derivative is highest), linear, inhomogeneous ($F_{ext}$ ) differential eq
  - The general solution to the inhomogeneous equation ($x_{gi}(t)$) is the general homogeneous ($x_{gh}(t)$) plus any inhomogeneous solution $x_{ai}(t)$ Thus: $x_{gi}(t) = x_{gh}(t) + x_{ai}(t)$.

## 12.3 The General Homogeneous Solution:. $x_{gh}(t)$ (video part 1, part 2)

- We first seek the most general homogeneous solution
- The homogeneous equation is: $m\, d^2x/dt^2 + b\, dx/dt + kx = 0$
  - The solution is of the form: $x_{gh}(t) = A\, e^{\alpha t + \delta}$ which we substitute into the above equation to get:
  - $(m\alpha^2 + b\alpha + k)\, A\, e^{\alpha t+\delta} = 0$ thus it follows that $m\alpha^2 + b\alpha + k = 0$ which is a quadratic eq.
  - Thus $\alpha = (-b \pm \sqrt{b^2 - 4mk})/(2m)$ or with $\gamma = b/2m$ and $\omega_0 = \sqrt{k/m}$ then we get
  - $\underline{\alpha = -\gamma \pm \sqrt{\gamma^2 - \omega_0{}^2}}$ is required for $x_{gh}(t)=A\, e^{\alpha t+\delta}$ as the general homogeneous solution.
- There are three types of solutions depending upon $\gamma$ and $\omega_0$ :
  - Overdamped: $\gamma > \omega_0$ then $x = Ae^{(-\gamma+\sqrt{\gamma^2-\omega_0^2})t} + Be^{(-\gamma-\sqrt{\gamma^2-\omega_0^2})t}$
  - Critically damped: $\gamma = \omega_0$ then $x = Ae^{-\gamma t} + Bte^{-\gamma t}$
  - Underdamped: $\gamma < \omega_0$ then defining $\omega = \sqrt{\omega_0^2 - \gamma^2}$ we get

    $x = Ae^{-\gamma t + i\omega t} + Be^{-\gamma t - i\omega t} = Ae^{-\gamma t}\cos(\omega t + \delta)$ where A and $\delta$ replace A & B as the constants
  - Description of each solution & the degenerate case
- This solution is called the transient solution as it dies out rapidly (like the initial static on a radio or TV).

### 12.4 The General Inhomogeneous Solution $x_{ai}(t)$ for a constant and an oscillatory force.

- Inhomogeneous force that is constant: $F = F_0$ is solved by adding $F_0/k$ to solution $x_{gh}(t)$
- Inhomogeneous oscillatory force $F = F_0 e^{i\omega_1 t}$ can be solved with $x_{ai}(t) = X e^{i\omega_1 t}$ for X:
  - Upon substitution we get $[m(i\omega_1)^2 + b(i\omega_1) + k]Xe^{i\omega_1 t} = F_0 e^{i\omega_1 t}$
  - Solving for X we get $X = (F_0/m) / ([ (i\omega_1)^2 + (b/m)(i\omega_1) + k/m]$ thus using $\gamma$ & $\omega_0$ we get:
  - $X = (F_0/m) / ((\omega_0^2 - \omega_1^2) + i2\gamma\omega_1)$ where we must put the complex number in normal form:
  - $(u+iv)^{-1} = (u-iv)/ (u^2+v^2)^{-1/2}$ which we put into the form $Re^{i\eta}$ with $R = (u^2+v^2)^{-1/2}$ and
  - thus $R = ((\omega_0^2 - \omega_1^2)^2 + (2\gamma\omega_1)^2)^{-1/2}$
  - $\eta = \tan^{-1}(-v/u) = \tan^{-1}(2\gamma\omega_1/(\omega_1^2 - \omega_0^2))$ where the '-' sign was put on the lower term.
  - This gives the final result that
  - $x_{ai}(t) = R e^{i\omega_1 t + i\eta}$
  - This solution is called the 'steady state' solution because it continues indefinitely

### 12.5 Discussion:

- Resonance can be easily seen as maximizing the amplitude R when $\omega_0 = \omega_1$
  - This occurs when the applied force is at the same frequency as the natural frequency $\omega_0$
  - Likewise one can see the phase shift $\eta$ between the response $x_{ai}(t)$ and the applied force.
- The general solution is then the sum of these two solutions $x_{gi}(t) = x_{gh}(t) + x_{ai}(t).+ F_0/k$
  - The homogeneous solution $x_{gh}(t)$ is called the transient as the term $e^{-\gamma t}$ decays with time.
  - The inhomogeneous solution is called the steady-state solution as it persists in time.

• The Most General Solution for Inhomogeneous Applied Forces

- The solution for any applied force can be obtained by Fourier Transforms with this solution

### 12.6 The general nature and importance of this result

- The problem of planetary orbits between two masses can be put in this form & same solution
- It also provides the general solution to an RLC circuit with a sinusoidal applied voltage.
- Thus these methods are of the greatest importance in physics.
- Using Fourier transforms, one can use these solutions to create the most general solution to any applied force in any form.

# 13 Fluids

## 13.1 Fluid Flow Described (video)

- A fluid is matter that flows and thus includes both liquids and gasses
- Fluid flow terms:
  - Steady Flow : the velocity is constant at each point in the fluid
  - Unsteady Flow: the velocity changes at a given point with time
  - Turbulent Flow: the velocity changes randomly and erratically in both magnitude & direction
  - Compressible: density of the fluid changes as pressure changes
  - Incompressible: the density of the fluid (essentially all liquids) is constant when pressure changes
  - Viscous Flow: Flow is impeded by loss of energy resisting the flow
  - Nonviscous Flow: Flow is smooth and non-resistive with no (or little) energy loss
  - Ideal Fluid = a Nonviscous incompressible fluid (water is a fair example)
  - Streamline Flow = The streamlines (trajectories of flow) are steady, constant velocity at one point

## 13.2 Density, Specific Gravity, (video)

- Mass Density per unit volume of a substance is defined by $\rho = m/V$ with units of $kg/m^3$
  - Examples of mass density: Brass 8470; Gold 19,300; Lead 11,300; Mercury 13600; Water 1,000
  - Also Wood 550; Ice 917; Aluminum 2,700; Air 1.29; Helium 0.18; Hydrogen 0.09; Oxygen 1.43
- Specific Gravity = Density of substance / Density of water at 4 degC (ie 1,000 $kg/m^3$)
  - Examples of specific gravity – e.g. what is the specific gravity of a human?

## 13.3 Pressure, and Archimedes Principle (video)

- Pressure is defined by $P = F/A$ with units of Pascal = Pa= $N/m^2$
  - Atmospheric pressure at sea level is 1.013E5 Pa ≈ 1E5 Pa
  - Pressure in a fluid $P = P_{surface} + \rho gh$ (derive $F= P_{sur} A + \rho(hA) g = P A$ then divide by A)
  - Pressure gauges (water & Hg columns supported)
  - Gauge pressure in a manometer: height is proportional to the difference of pressures
  - Pascal's principle: pressure applied to an enclosed fluid is transmitted to all parts
    - ◊ $F_1 / A_1 = F_2 / A_2$ can be used to lift a heavy object (car) as a hydraulic lift
- Archimedes' (287-212 BCE) Principle: $F_{buoyant} = W_{fluid\ displaced}$

## 13.4 Fluid Flow Equations (video)

- Equation of Continuity relates the mass flow rate at two points in the fluid
  - Is equivalent to the conservation of mass
  - $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$ (ie is conserved from one point to another)
  - Derive: ****
- Bernoulli's (1700-1782) Equation governs the steady nonviscous incompressible fluid flow
  - Is equivalent to conservation of energy
  - $P_1 + 1/2\rho v_1^2 + \rho g y_1 = P_2 + 1/2\rho v_2^2 + \rho g y_2$ (ie is conserved from one point to another)
  - Derive: ****

## 13.5 Viscous Flow (video)

- Viscous Flow describes the Force needed to move a layer of viscous fluid at constant velocity
  - $F= \eta Av / y$ where $\eta$ = the coefficient of viscosity (units of Pa*s, also 1 poise = 0.1 Pa s)
    - ◊ where A is the area of the fluid, v is its velocity, and y is distance from immovable plane
- Poiseuille's law gives the volume flow rate Q in a pipe of radius R, length L, and pressures $P_1$, $P_2$
  - $Q = dV/dt = \pi R^4 (P_2 - P_1) /(8\eta L)$

# 14 Mechanical Waves & Sound

## 14.1 Definition of waves and Key Concepts (video)

- A wave is a traveling disturbance in a media that carries energy but not mass
- Fourier's theorem
  - All wave disturbances are (linear) combinations of sin & cos waves of different freq
- Core concepts concerning waves
- General Wave Equation for wave displacement (y) is: $y(x,t) = A\cos(\omega t - kx + \delta)$
  - The period, T, which is the time required for one full cycle of the wave
  - The frequency, f, is the number of compete cycles per unit time (second):
    - Frequency units are: Hertz = Hz =Cycles/s
  - The amplitude, A, of the wave is the maximum displacement from equilibrium
  - The wavelength, $\lambda$, is the (shortest) length between two identical parts of the wave
  - The phase, $\delta$, of a wave is the angle in radians that the wave is displaced in sin or cos
  - The angular velocity $\omega = 2\pi f$ (derive from $\omega T = 2\pi$ )
  - The wave number $k = 2\pi / \lambda$ (derive from $k\lambda = 2\pi$ )

## 14.2 Sound Waves (video)

- Objective (physically measurable) aspects of sound verses
  - Subjective (perceived by human senses)
- Intensity of the wave (in Watts / $m^2$) verses Loudness (measured in decibels)
  - Loudness is measured in decibels (dB) $\beta = 10\log(I/I_0)$ where I = intensity in $w/m^2$,
    - $I_0 = 10^{-12}$ $w/m^2$ is the threshold of human hearing
    - An increase if 10 dB is perceived as twice the loudness
- Frequency (Hz) verses the perceived frequency or Pitch
  - Musical frequency: A above middle C is 440 Hz and is the standard of western music
  - The standard for acoustics and sound for human hearing is 1,000 Hz = 1KHz
  - The normal maximum range of human hearing is 20Hz to 20KHz
    - Bats can hear up to about 120 KHz – What living thing can hear a higher freq?
- Velocity of important waves:
  - Velocity of sound is 331 m/s at 0 C and increases by 0.6 m/s for each degree C
    - Velocity of sound is also about 1100 ft/s which is about 2 city blocks
  - V of sound in substances m/s: Steel 5,960; Glass 5,640; Water 1,482; Helium 965
  - Velocity of light = c = 3E8 m/s (aprox) in a vacuum (discussed later with lenses)
  - Velocity of a wave on a string = $v_{string} = (F / (m/l))^{1/2}$ (m/l) = the mass per unit length
- Harmonic Structure (composition of overtones or harmonics) verses the Quality
  - Aspects of overtones enables one to distinguish instruments and voices

## 14.3 Critical Wave Equations (video)

- Frequency (f) – Wave Length ($\lambda$) – Velocity (v) relationship: $f\lambda = v$ for ANY wave
- Frequency (f) Period (T) relationship: $f = 1/T$
  - Example of a radio wave: f = 102 MHz, c = 3E8 m/s thus $\lambda$ = 1.02E8/3E8 = 0.34 m
  - Example of a sound wave: f = A 440 Hz, vsound = 1100 ft/s thus $\lambda$ = 2.5 ft
  - Examples

## 14.4 Doppler shift (video)

- The Doppler shift in frequency results when a source is moving $v_s$ or the observer at $v_o$
  - Observer moves toward source: $f_0 = f_s (1-v_0/v)/(1+v_s/v)$
    - Away from source $f_0 = f_s (1+v_0/v)/(1-v_s/v)$

## 14.5 The Logarithmic Nature of Responses to Stimulation: (video)

- NOTE: The human body responds to sound intensity, frequency, light intensity, heat, pressure and other stimulations as the log of the stimulus. This allows a person to have a vast range of sensing without overloading the senses at high values and still be extremely sensitive to low values.
  - E.G. sound intensity is measured in $\log(I/I_0)$ and the piano scale is log of the frequency
  - Consider the fact that information is measured as the logarithm of a probability

- Perhaps life forms take the sensory log to automatically measure the maximum information?

# 15 Linear Superposition of Waves, Interference, & Music

## 15.1 Linear Superposition (video)

- Linear Superposition: The total wave amplitude at a point is the sum of the separate waves
  - Constructive Interference:
    - When both waves are of the same sign & become greater than each separately
  - Destructive Interference:
    - When the two waves are of opposite signs and thus partly cancel each other
  - If a wave proceeds by two paths:
    - the phase difference due to path length can be constructive or destructive
- Importance of linear superposition in physics & science
  - Waves, electric and magnetic fields, gravity, forces allows the sum of separate fields
    - or forces to be computed from the separate components,
- Direction of wave vibration relative to motion distinguishes two types of waves:
  - Transverse waves: where the media vibrates perpendicular to the velocity
    - e.g. EM waves including light as E & M are orthogonal to v &
      - surface water waves
  - Longitudinal waves: where the media vibrates parallel to the velocity
    - e.g. sound (compression) waves
  - Torsion waves, a third type, is very rare and consists of a twisting wave about v axis

## 15.2 Wave Interference (video part 1, part 2)

- Interference occurs between a wave and itself dependent upon the paths taken $\Delta x$ :
  - Constructive interference: $\Delta x = n\,\lambda$ where n = 0, 1, 3, ...
  - Destructive interference: $\Delta x = (n+1/2)\,\lambda$ where n = 0, 1, 3, ...
- Interference of a single slit of width D:
  - Angle to respective maxima is $\sin\theta = \lambda/D$ ($=1.22\,\lambda/D$ circular)
- Interference of two nearby frequencies $f_1$ & $f_2$ results in the average frequency with beats:
  - One hears $½(f_1 + f_2) * ½\,(f_1 - f_2)$ = average frequency * beats with frequency $½\,(f_1 - f_2)$
    - These 'beats' are really modulations (oscillations) in the amplitude of the average freq.
  - Since the 'frequency' $½\,(f_1 - f_2)$ has two maxima per cycle, one gets a beat period of $T=1/(f_1 - f_2)$
  - This can be used to tune one instrument using another as a standard

## 15.3 The Foundation of strings and horns for musical instruments (video)

- String (and air column) vibrations:
- Stretched strings of length L sustain vibrations that have an integer number of half waves in L
  - Thus with a node at each end (the attached point cannot move) we get $n(\lambda/2) = L$
  - Thus the frequencies for each integer n is given by:
    - $f_n = v/\lambda = n\,v/(2L) = n\,f_1$ thus multiples of $f_1$

## 15.4 Air Columns (video)

- Air columns that are closed at both ends have nodes there and thus obey the same equation.
  - If an air column is open at one end, one has an antinode thus $(n_{odd}/4)\lambda = L$
  - Thus : $f_n = v/\lambda = n_{odd}\,v/(4L) = n_{odd}\,f_1$ where $n_{odd}$ = 1, 3, 5, 7, ….
- Harmonics and Overtones
  - These values of n refer to the 'n$^{th}$' harmonic or the (n-1)$^{th}$ overtone
    - where n=1 is fundamental
  - Thus the 5$^{th}$ harmonic is 4$^{th}$ overtone; and the 1$^{st}$ harmonic is the fundamental.

## 15.5 Musical Frequencies: (video)

  - Two notes sound 'consonant' when their frequencies are nice integer multiples
    - Discovered by Pythagoras
  - Unison is 1/1, an octave is 2/1, a fifth is 3/2; and a fourth is ¾ in order of consonance
  - When a string is plucked or air column sounded
    - the frequencies = integers times the fundamental
  - Pythagoras tuned early instruments by going up a fifth, down a fourth, up a fifth, etc

## 15.6 Bach Equitempered Tuning (video)

- An improved method was invented by JS Bach
  - called equitempered tuning (all half steps equal)
- For the 12 half steps in an octave in western music, each half step goes up by a factor $\alpha$
- Thus the notes are $f_1, \alpha f_1, \alpha^2 f_1, \ldots \alpha^{12} f_1$ which must = 2 $f_1$ (an octave)
  - thus $\alpha = 2^{1/12} = 1.05946$
- This is the ratio of two adjacent notes a half step apart in music.
- The standard that fixes all the notes is $A_{440}$ = 440 Hz which is the A above middle C
- In principle, one can now compute the frequency of every note in western music.

- Perfect frequency ratios & the Equitempered value:
  - Fifth (3/2 , 1.49831), Fourth (4/3, 1.33484),
  - Maj Third (5/4, 1.25992), Min Third (6/5, 1.18921),
  - Maj Six (5/3, 1.68179), Min Six (8/5, 1.58740)

## 15.7 Advanced aspects of acoustics and music (video)

- Just discernable differences in frequency. At 1,000 Hz & higher one can discern a 0.5% freq change
  - A 'cent' = 1/100 of a half step. One can discern a frequency difference of about 5 cents.
  - Just as a half note ratio is $2^{1/12}$, the cent is the ratio $2^{1/1200} = 1.00057779$
- Just discernable differences in loudness, although varying with freq etc, is about 1.0 dB
- Differences between the equitempered frequencies and 'just' or 'perfect ratios of intervals

## 15.8 Acoustical Reverberations (video)

- Reverberation Time = Time for the sound intensity level to reduce to 1E-6 (60dB) of original value
  - $T(s) = 0.049\ V/A$ where V ($ft^3$) = volume of the room and A = area of an absorbing 'hole' ($ft^2$)
  - The perfectly absorbing hole area, $A = \Sigma a_i S_i$ where $a_i$ is the absorption coef. of an area of $S_i$ $ft^2$
  - Approximate optimal T values in sec are: Speech 0.4 to 0.8; music 1 to 1.6, etc
  - Absorption values at 1kHz are $a_i =$
    - Marble 0.01; Plate glass 0.04; Plywood on studs 0.10; Carpet 0.37;
    - Plaster 0.10; Acoustical plaster 0.78; Each person 7.0; Empty cloth seat 5.0

# 16 Temperature & Heat

## 16.1 Temperature and Heat Defined (video)

- Temperature: a measure of the average random energy in a substance.
- Units: temperature scales
  - Fahrenheit scale:
    - 0 ˚F: freezing sea water, 100 ˚F: for human body, then 32˚F: freezing water
  - Celsius scale:
    - 0 ˚C: freezing water, 100 ˚C: for boiling water then -273.15 = absolute zero
  - Kelvin scale: by definition
    - ˚K = 273.15+˚C.
  - All scales are defined in terms of ˚K where
    - 0˚K is absolute zero & 273.16 ˚K = water triple point
- Thermometers
  - Primarily use the 'linear' expansion of a substance such as mercury with temperature
  - Optimal thermometer is the constant volume gas thermometer of an 'ideal gas'
- Temperature conversion: F = 32 +C*9/5, C = (F-32)*5/9, K = C + 273.15
- Heat is random (mostly kinetic) energy in a substance – the energy that flows due to temperature diff.
  - The standard SI units of heat is the Joule (J) as it is the SI unit of energy in general
  - Also: 1 Calorie = amt of heat needed to raise the temperature of 1 kg of water 1 C
    - The Calorie (upper case) = 1000 calories which pertain to a gram of water not kg
    - It is the Calorie or Kilocalorie that we eat when we eat food (energy)
  - Also 1 BTU = amt of heat needed to raise the temperature of 1 pound of water 1 F

## 16.2 Expansion of Heated Substances (video)

- Linear thermal expansion of a solid:
  - Change $\Delta L$ in length $L_0$ due to a change $\Delta T$ in temperature is
  - $\Delta L = \alpha L_0 \Delta T$ where $\alpha$ is the coefficient of linear expansion in 1/C
  - Examples of $\alpha$ are : Brass 19E-6; Gold 14E-6; Glass 8.5E-6; Aluminum 23E-6
- Volumetric Expansion of a solid or liquid: Change $\Delta V$ in length $V_0$ due to a change $\Delta T$ in temperature
  - $\Delta V = \beta V_0 \Delta T$ where $\beta$ is the coefficient of volume expansion in 1/C

## 16.3 Addition of Heat Can Raise the Temperature: (video)

- Heat raises the temperature of a substance (except during a phase change) by :
  - $Q = c\, m\, \Delta T$ where c is the specific heat of the substance
  - Examples of c (J/(kg C): Water 4186; Mercury 139; Aluminum 900; Glass 840; Lead 128;
- The heat Q required for a phase change is $Q = m\, L$
    - where m = mass and L is the latent heat
  - Latent heat of fusion, $L_f$, refers to melting or freezing (J/kg)
  - Latent heat of vaporization, $L_v$, refers to boiling or condensation (J/kg)
  - $L_f$ & $L_v$ in (J/kg): Water 33.5E4, 22.6E5; Gold 6.28E4, 17.2E5; Nitrogen 2.60E4, 2.00E5
  - $T_{melt}$ & $T_{boil}$ in Celcius: Water 0, 100; Gold 1063, 2808; Nitrogen -210.0, -195.8

# 17 Transfer of Heat

## 17.1 Transfer of Heat by Conduction (video)

- Conduction: heat is transferred through a material without motion of the material itself
  - Distinguish thermal conductors from thermal insulators
  - The formulas for conduction in solids is simple and of great importance
  - Conduction heat/time $\Delta Q/\Delta t = k\ A\ \Delta T\ /\ L$
    - where k= thermal conductivity, A=area, L=thickness T=temp
  - Thus: $\Delta Q/\Delta t = A\ \Delta T\ /\ (L/k) = A\ \Delta T\ /\ R$
    - where $R = L/k$ is called the R factor (combines k & L)
  - R factors are additive for building materials
    - and with normal US units of BTU/hr for $\Delta Q/\Delta t$, and A in $ft^2$, T $^\circ$ F
  - Values are:
    - R = 1 glass, 2 double pane; R=11 for 3.5" wall insul, R=19 for 6" floor/attic insul
    - and R= about 3.4 for uninsulated walls, floors, and ceilings .
  - Problems involving building materials allow the R factors to simply add to obtain the total.

## 17.2 Transfer of Heat by Radiation: (video)

- Radiation: the process by which electromagnetic radiation (cavity radiation) is emitted
  - The profile of emitted radiation is dependent upon the temperature of the object
  - We are familiar with substances that emit infrared (heat) because they are hot
  - We are also familiar with much hotter objects that glow red hot, or white or even blue.
  - The formula for radiation is also relatively simple but unusual.
- Radiation (Stefan Boltzman law): $\Delta Q/\Delta t = \varepsilon\ \sigma\ A\ T^4$
  - where $\varepsilon$ is emissivity ($\varepsilon$ =1 black, 0 shiny metal) and where
  - $\sigma$ = Stefan Boltzman constant = 5.67051 E-8 (J/(s $m^2$ $K^4$)), and A is the area in $m^2$

## 17.3 Transfer of Heat by Convection (video)

- Convection: the process of conveying heat from one point to another by the movement of fluid
  - Distinguish natural convection or forced convection
  - The formulas for convection are extremely complex and nonlinear as they are fluid flows
  - So at this level we do not attempt to discuss the mathematical aspects of convection

# 18 Ideal Gas Law & Kinetic Theory

## 18.1 Define the Mole, AMU, & Avogadro's number (video)

- Atomic Mass Unit = 1.6605402E-27 kg =1/12 of the mass of $^{12}C$ (as this is the best reference)
  - Previously the hydrogen atom was taken as '1 amu' but C is more accurate.
- Mole = the number of entities equal to the number of atoms in 12 grams of $^{12}C$
  - Mole = Avogadro's number = $N_A$ = 6.0221367 E23
  - Avogadro's number of entities (ie one mole) of a chemical is its molecular mass in grams
  - Thus 18 grams of $H_20$ is one mole and contains $N_A$ molecules
- An ideal gas is low density, point particles with no internal freedoms, and elastic collisions
  - A perfect ideal gas is helium as it has a completed outer shell and forms no compounds.

## 18.2 Ideal Gas Law (video)

- Ideal gas law: $P V = n R T$ (P=Pressure, V=Volume, n= number of moles, T = temp. in ° K )
  - where R is the Universal Gas Constant 8.314510 J/(mole*K)
- Equivalently one can write $PV = (n^* N_A) (R/ N_A) T = N k T$
  - where N = Number of molecules and
  - $k = R/ N_A$ the Boltzman constant = 1.380658 E-23 J/K
- Historical Origin was in other discoveries:
  - Boyles law (constant T) gives $P_1V_1 = P_2V_2$ used to compare a gas 'before and after'
  - Charles law (constant P) gives $V_1 /T_1 = V_2/T_2$
- Ideal gas law as derived from kinetic theory:
  - Kinetic theory shows: $PV = (2/3) N \langle KE \rangle$ thus when combined with the ideal gas law
  - Thus the average kinetic energy is $\langle KE \rangle = (3/2) k T$ thereby interpreting temperature
- Also the internal energy $U = N \langle KE \rangle$ thus $U = (3/2) N k T = (3/2) n R T$ for a monoatomic gas

## 18.3 Diffusion: Irreversible process of Increasing Entropy (video)

- Diffusion – Fick's Law of Diffusion:
  - $\Delta m/\Delta t = (D A \Delta C) / L$ = mass per time diffusing in a solvent
  - where $\Delta C$ is the concentration difference, in a channel of length L & cross section area A
  - The diffusion constant D for water vapor in air is 2.4E-5 $m^2/s$

## 18.4 Derivation of the Relationship of T to Average Kinetic Energy

# 19 Thermodynamics

## 19.1 The Four Laws of Thermodynamics Described (video)

- Laws of thermodynamics:
  - $0^{th}$ law: Two systems in equilibrium with a third system are in equilibrium with each other
  - $1^{st}$ law: The change in internal energy is equal to the heat gained minus the work done
    - This is the law of conservation of energy including heat in the equation
  - $2^{nd}$ law: Heat flows spontaneously from a higher T to one of lower T, never conversely
    - or: The total entropy (disorder) always increases for an irreversible process
      - and entropy is constant for a reversible process.
  - $3^{rd}$ law: It is not possible to lower system temperature to absolute zero in a finite number of steps
- Types of named processes
  - Isobaric means that pressure is kept constant ($\Delta P = 0$)
  - Isothermal means that temperature is kept constant ($\Delta T = 0$)
    - Examples with weather maps
  - Isochoric (or isovolumetric) means that the volume is kept constant ($\Delta V = 0$)
  - Adiabatic process is one in which there is no change (flow) of heat ($\Delta Q = 0$)

## 19.2 Mathematical Aspects of the First Law (video)

- $1^{st}$ Law of thermodynamics:
  - The $1^{st}$ Law: The change in internal energy = $\Delta U = \Delta Q - \Delta W$ where
    - $\Delta Q$ is the heat input into the system and $\Delta W$ is the work done by the system
    - This form originates with steam engines where heat is input and work is extracted
  - For isobaric process ($\Delta P = 0$), the work done is $\Delta W = P \Delta V$
  - For isothermal quasi-static ideal gas process $\Delta W = n R T \ln(V_f/V_i)$
  - For adiabatic ($\Delta Q = 0$) quasi-static process $\Delta W = (3/2) n R (T_i - T_f)$ for n moles of monoatomic gas
  - Also for an adiabatic ideal gas: $P V_i^{\gamma} = P V_f^{\gamma}$ where $\gamma = c_p / c_v$

## 19.3 Specific heat capacities for gasses: (video)

- Recall $\Delta Q = C \Delta T$ where C is the specific heat:
- $C_P = (5/2) R$ for a monatomic ideal gas at constant pressure and
  - $C_v = (3/2) R$ at constant volume
- $C_P = (7/2) R$ for a diatomic ideal gas at constant pressure and
  - $C_v = (5/2) R$ at constant volume
- For any type of ideal gas $C_P - C_P = R$

## 19.4 Efficiency of Heat Engines (video)

- Heat Engines take in heat Q and output useful work W with an efficiency $\varepsilon = W/Q$
  - but since $Q_h = W + Q_c$ then $\varepsilon = W/Q_h = 1 - Q_c/Q_h$ (all terms are positive magnitudes)
  - For a Carnot engine: $Q_c/Q_h = T_c/T_h$ thus $\varepsilon_{carnot} = 1 - T_c/T_h$
    - Theoretical Best Efficiency for power plants: $T_h = 750$ K, $T_c = 300$K
      - (Note that $T_c = 300$K is the standard temperature for Earth)
    - Thus $\varepsilon_{carnot} = 1 - 300/750 = 0.60$ or 60% efficient but $\varepsilon_{actual} = 0.40$ or 40%
    - Thus 60% of all energy generated by a power plant goes is wasted as heat.
- Coefficient of Performance (COP) for refrigerators and heat pumps:
  - $COP_{ref} = Q_c / W$ and $COP_{hp} = Q_h / W$

## 19.5 Mathematical Aspects of the Second Law (video)

- Entropy changes $\Delta S$ in which heat enters or leaves a system reversible at constant T is::
  - $\Delta S = \Delta Q/T$
  - Entropy is a measure of the system disorder
  - Problem: Compute the entropy change of melting ice

# 20 Electric Forces

## 20.1 Fundamental Terms for Electrical Charge, Conductors, & Insulators (video)

- We are all familiar with static electricity, lightning, and electrical currents from an early age.
- We are familiar with sources of charge: electrons, protons, ions, and atomic structure.
  - What is electrical charge? We do not really know – it is an intrinsic property like mass.
- Electric charges are + & - Like charges (++ and - -) repel while opposites (+ -) attract.
  - Benjamin Franklin (1706-1790) defined charge & related it to lightning
- Charges are quantized in integer multiples of the basic charge e = 1.6E-19 C
  - Robert Milliken proved this in 1909 and measured the charge on the electron $e^-$
- Electric charge is measured in units of Coulombs
- The total electric charge in a closed domain is conserved
- Conductors allow charges to move freely. Other materials are called insulators.
- Electric Induction Charging – a conductor attached to the ground is 'grounding'
  - Contact charging is when a charged object touches a neutral object & leaves it charged
- Linear Superposition: electrical (&magnetic) forces are (vectorially) additive from individual forces

## 20.2 Coulomb's Law (video)

- Coulombs law discovered 1785 By Charles Coulomb using a torsion balance to determine $F_c$
- Coulombs Law for forces between charges:
  - $F_{1\text{-}2} = k_e q_1\, q_2 / r^2$ where $k_e$ = 9E9 = $1/(4\pi\varepsilon_0)$ exactly = 8.9875 E9
  - The constant $\varepsilon_0$ is the permittivity of the vacuum
  - Force F is measured in Newtons
- Charge per unit volume $\rho = Q/V$, per unit area $\sigma = Q/A$, & per length $\lambda = Q/l$
  - Problems with two charges
  - Vector problems with multiple charges

## 20.3 Formal Vector Form of Coulomb's Law and with Multiple Charges (video)

- Vector Statement of Coulomb's Law
  - $\mathbf{F}_{1\to 2} = k_e q_1\, q_2\, (\mathbf{r}_2\text{-}\mathbf{r}_1) / |\mathbf{r}_2\text{-}\mathbf{r}_1|^3$ where **F** and **r** are vectors
- Generally the force on a charge q from other charges is $\mathbf{F}_q(r) = q\, \Sigma_I\, q_i\, (\mathbf{r}\text{-}\mathbf{r}_i) / |\mathbf{r}\text{-}\mathbf{r}_i|^3$ thus:
- Vector problems

# 21 Electric Field

## 21.1 Description and Origin of the Electric Field Concept (video)

- Force at a distance was difficult for people to accept – thus the electric field, **E**, was 'invented'
- The electric field at a point is the force a unit charge would experience. Show field lines.
- **E**(x,y,z,t) is a vector field. Describe a vector field – like wind velocity on a weather map
- Electric field lines display E . (E was at first an imaginary concept.)
  - They can never cross. They begin at + and end at – charges.
  - E is zero inside a conducting material and excess resides on the surface.
  - E just outside a conductor is always perpendicular to the conductor's surface.
- Charge accumulates where the surface has the smallest radius of curvature.
  - On a conductor, charge accumulates where the radius of curvature is the smallest.
  - **E** of a charged sphere shell is as though all charge is at its center (outside the sphere)
  - The electric field of a charged spherical shell is zero (inside the sphere) - shielding
- The electric field inside a parallel plate capacitor is uniform & often used as a source of an E field.

## 21.2 Definition of the Electric Field (video)

- $\mathbf{E} = \mathbf{F}/q = kq_0/r^2$ thus $\mathbf{F} = q\,\mathbf{E}$
  - **More generally;** $\mathbf{E}(r) = \Sigma_i\, q_i\, (\mathbf{r}-\mathbf{r}_i) / |\mathbf{r}-\mathbf{r}_i|^3 = \mathbf{F}_q(r)/q$
  - E has units of Newtons / Coulomb (there is no special name for this unit)
- Examples and problems of point charge and multiple charges adding **E** vectors
- Motion of a charged particle in a constant E field. $m\mathbf{a} = q\mathbf{E}$, use "constant a" formulas
- Exact vector formula: $\mathbf{E}(\mathbf{r}) = k\; q_1\, (\mathbf{r}-\mathbf{r}_1) / |\mathbf{r}-\mathbf{r}_1|^3$ where **E** and **r** are vectors
-

## 21.3 Electric Dipoles (video part 1, part 2)

- The electric field of a dipole (+ -)
- Electric dipole is a pair of equal but opposite charges separated by a distance
  - Some molecules are dipolar such as water
  - The electric field of a dipole is similar to that of a magnetic dipole (magnet).
  - Draw the field lines of a dipole
- Electric dipole moment **p** is defined as $\mathbf{p} = Q\mathbf{d}$ where +Q and –Q are a distance d apart
  - The electric dipole **p** is a vector pointing along d from the negative to the positive charge

## 21.4 Torque and Potential energy of a Dipole in an Electric Field (video)

- An electric dipole feels a torque in an electric field of $\tau = \mathbf{p} \times \mathbf{E}$ where $\tau$ is a vector
- An electric dipole in a field E has an energy of $U = -\mathbf{p} * \mathbf{E}$ where U is a scalar
- Examples and problems with dipoles in an electric field

# 22 Gauss' Law

## 22.1 Flux of the Electric Field and Gauss' Law (video)

- The flux of a vector field, V, through a surface of area A is $\Phi = V * A$
- Gauss' law states that the flux of the electric field through a closed surface is $\Phi = q_{inside} / \varepsilon_0$
- A more formal vector calculus equation for the flux allows us to prove this from Coulombs law
  - With $\Phi = \int \mathbf{E} \bullet d\sigma = (q/4\pi\, \varepsilon_0\, r^2) * (4\pi\, r^2)$ we get $\Phi = q / \varepsilon_0$
  - Gauss' law can be used to compute the electric field in symmetric cases.
    - The electric field is zero everywhere inside a conductor thus conductors can be used to shield the inside region from an outside electric field.
    - Any excess charge resides on the surface of the conductor
  - Do simple flux calculation for several charges inside a closed surface

## 22.2 E Calculations using Gauss' Law (video)

- Plane: $E = \sigma / (2\varepsilon_0)$
- Line charge: $E = \lambda / (2\pi\varepsilon_0 r)$
- Inside a parallel plate capacitor: $E = \sigma / (\varepsilon_0)$ and is uniform
- $E = \sigma / \varepsilon_0$ = Also just outside a conductor
- Exercises: Derive these

# 23 Electric Potential & Potential Energy

## 23.1 Introduction (video)

- The potential energy of a system is the work necessary to assemble them from infinity
  - The potential energy, U, is a scalar and is measured in units of Joules
- The electric potential V(r), is the work needed to bring a unit charge to this point from infinity
  - V(r) is also a scalar and is measured in units of Volts = Joule / Coulomb
- The electric equipotential lines are like isotherms, isobars (pressure), or gravity potential
  - The plotting of the equal potential lines V(r)=constant for a system displays contours of V
  - These contours are always exactly perpendicular to the electric field **E** lines everywhere
    - Note that the potential (like the electric field) exists at every point
    - Potential is a property of the position whether anything is there or not.
  - E is equal to (the negative of ) the gradient (rate and direction of maximum change) of V
  - Constant V(r) curves are good visual representations of the electrostatic environment
  - Always use changes in V (voltage differences) rather than absolute values
    - Convince yourself that the constant part of the potential is not observable.

## 23.2 Mathematical Form of Potential Energy: (video)

- Potential Energy = $U = k\, q_1\, q_2 / |r_1 - r_2|$ = Work needed to bring $q_1$ & $q_2$ from an infinite distance
  - The units of potential energy here are Joules. Note that U is a scalar not a vector.
  - The potential energy of several charges, $q_i$ is given by $U = \frac{1}{2}\ k\, \Sigma q_i\, q_j / |r_i - r_j|$
    - note the ½ arises from double counting in the summation over i and j
  - Examples

## 23.3 Mathematical Form of the Potential Function: (video)

- Electric Potential = $V(\mathbf{r}) = U/q_0$ = the work needed to bring a unit charge $q_0$ from infinity to the point **r**
  - Thus $V(\mathbf{r}) = k\, q / r$ at **r** due to a charge q at the origin
  - The units of electric potential are given in Volts = Joules / Coulomb (or V=J/Q)
  - Usually, we look at voltage differences such as the potential difference between battery terminals.
  - Examples of different types of batteries (1.5 V for AA AAA C D and 12 V for autos)
- Equipotential lines (curves that follow equal potential values) are perpendicular everywhere to **E**

## 23.4 Vector Calculus Mathematical Form: (video)

- Potential Energy = Work = $dU = \mathbf{F} \bullet d\mathbf{r} = -q_1 \int \mathbf{E} \bullet d\mathbf{r} = -k\, q_1\, q_2 \int dr_{12} / r_{12}^2$
  - Thus $U = k\, q_1\, q_2 / r_{12}$ where $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$ and when the integral goes from infinity up to $r_{12}$
  - The units of potential energy U are in Joules and U is a scalar as it is a dot product
- Electric Potential = $V = U/q$ or for a single charge at the origin, $V(r) = k\, q / r$
  - The units of V are in Volts (V) where V=J/Q
  - Since $\Delta V = -\int \mathbf{E} \bullet d\mathbf{r}$ then it follows that $E_x = -\partial V / \partial x$ and generally that $\mathbf{E} = -\nabla V$
  - One recalls that $\nabla \equiv (\partial / \partial x, \partial / \partial y, \partial / \partial z)$

# 24 Capacitance

## 24.1 Definition of Capacitance (video)

- Take a charge Q from object A to object B, (both neutral objects) then
  - A potential difference of V volts between A and B will result from this action.
  - The more charge one carries from A to B then the greater the voltage will be.
  - If 2Q, 3Q etc is moved from A to B then 2V, 3V etc will be the resulting voltage difference.
  - This constant ratio of Q/V depends upon the geometry and is defined as the capacitance
- Capacitance for a system is defined as: $C = Q / V$ <Farad (F) = Coulomb / Volt >
  - Capacitors were the earliest methods of storing charge, voltage, and electrical energy.

## 24.2 Types and Combinations of Capacitors (video)

- Capacitance of a parallel plate capacitor
  - $C = q/V = \sigma A / (Ed) = \sigma A / ((\sigma/\varepsilon_0)d)$ or $C = \varepsilon_0 A/d$
- Combinations of capacitors:
  - In parallel $C_{total} = C_1 + C_2 + \dots C_n$
  - In series $1/C_{total} = 1/C_1 + 1/C_2 + \dots 1/C_n$
- Energy stored in a capacitor $W = ½ \; Q V = ½ \, C V^2$

## 24.3 Dielectric materials (video)

- If a <u>dielectric material</u> is placed in a capacitor then $V=V_0/\kappa$
- Where $\kappa$ = dielectric constant $\kappa = 1$ for vacuum or air, 3.7 paper, 80 water…
- It follows that $C = \kappa C_0$ since the charge is unchanged on the capacitor.

## 24.4 Capacitance values for other simple geometries: (video)

- Recall that the capacitance of parallel plates of area A and separation d is $C = \varepsilon_o A /d$
- A charged sphere of radius R: $C = 4\pi\varepsilon_o R$
- Cylindrical capacitor of length l and inner & outer radii a & b : $C = l / [2 k \ln (b/a)]$
- Spherical capacitor of inner and outer radii a & b: $C = ab /[k (b-a)]$

# 25 Electric Current & Resistance

## 25.1 Electric Current (video)

- When a potential difference (voltage) exists across a substance, the charges try to move to equalize it and thus a flow of electrical charge results called an electric current.
- Electrical current is the amount of charge in Coulombs that flows per second past a point
- Thus electric current is defined as = $I = \Delta Q / \Delta t$
    - The unit of electrical current is the Ampere = Coulomb / Second or A =C/s
    - Problems

## 25.2 Electrical Resistance (video)

- There is resistance to all flow of electrical current (except in superconductors).
    - The ratio of the voltage to the current that flows, is a constant called the resistance
    - The constancy of the ratio of voltage to current is Ohm's law – an experimental result.
- Ohm's law states: $R = V/I$ <Ohm = Volt / Ampere $\Omega = V/A$ > $V = IR$
- Resistors in series & parallel:
    - Resistors in series: $R_{series} = R_1 + R_2 + R_3 + \ldots$
    - Resistors in parallel: $1/R_{parallel} = 1/R_1 + 1/R_2 + 1/R_3 + \ldots$
    - Problems

## 25.3 Resistivity – the inherent resistance for a given substance (not for an object) (video)

- One can show that $R = \rho\, l / A$ where $\rho$ is defined for a given material – the resistively
    - Examples of values $\rho_{silver} = 1.59\text{E-}8$ $\rho_{copper} = 1.72\text{E-}8$ $\rho_{aluminum} = 2.82\text{E-}8$
    - $\rho_{iron} = 9.7\text{E-}8$ $\rho_{carbon} = 3.5\text{E-}5$ $\rho_{wood} = 3\text{E}10$ $\rho_{glass} = 10^{10}$ to $10^{14}$
- $\rho$ depends upon temperature: $\rho = \rho_0 (1 + \alpha \,\Gamma (T-T_0))$ and thus increases with temperature.
- Electrical conductivity of a substance $\sigma = 1/\rho$

## 25.4 Power (energy) loss (video)

- Power Loss $P = IV = I^2R$
- Consequently for long distance power lines it is optimal to minimize the current due to the square

## 25.5 Current Density (video)

- Electric current density $j = I / A = n\, q\, v$
- Ohms law with current density $j = \sigma E$ where $\sigma$ = conductivity
    - Proof: $V = IR = I \rho L / A = (I/A)(L/\sigma) = j (L/\sigma)$
    - Thus $j = \sigma (V/L) = \sigma E$
- The current density is a more general concept that can be studied at different points in space
    - (not just for an object)

# 26 Direct Electrical Currents

## 26.1 Kirchhoff's Laws: (video)

- Sum of currents entering a junction must equal the sum leaving the junction (node)
- Sum of voltages across each element in any closed loop must be zero.
  - Examples
- Discuss:
  - Voltmeter
  - Galvanometer
  - Ammeter
- Discuss household wiring 110V and 220V, circuit breakers, …
  - Examples
-

## 26.2 RCV Circuit (video)

- Kirchhoff's second law gives the equation: $RI + q/C = V$ or $R(dq/dt) + q/C = V$
  - This is a first order (first derivative is highest), linear (only first power of q and derivative)
    - ◊ differential (has derivities but no integrals) equation.
  - It is also inhomogeneous because of the constant V but if V=0, would be homogeneous.
- The solution to all linear differential equations is of the form $q(t) = Ae^{\alpha t} + B$
  - Show conditions for solution which gives $\alpha = -1/RC$ and $B = Q(0)$
- RCV circuit : $\tau = RC$ is called the time constant of the circuit
  - Note that RC has the units of time and gives, in t=RC, the factor $e^{-1} = 1/e = 1/2.71828$
- If charging from a voltage V applied at t=0 then $q(t) = Q_0(1-e^{-t/RC})$ and $i(t) = (V/R)e^{-t/RC}$
  - ◊ where $Q_0 = CV$
- If discharging a charged capacitor from t=0 then $q(t) = Q_0e^{-t/RC}$ and $i(t) = I_0e^{-t/RC}$
  - ◊ where $Q_0$ = initial charge on the capacitor, and $I_0 = Q/RC$

# 27 Magnetic Fields

## 27.1 Magnetic Fields from Natural Objects and the Environment (video)

- In early science classes we play with magnets & learn about the N & S poles
  - Like poles (NN & SS) repel and unlike poles (NS) attract
  - With iron filings on paper over a magnet, one sees the alignment of the magnetic 'field' B
- The earth's magnetic field was discovered with the orientation of certain rocks:– lode stones
  - These were a technological breakthrough used for early ocean navigation
- We define the pole that points to the earth's geographical 'North Pole' as the magnetic N pole
  - Thus actually under the earth's geographical North Pole, there is a magnetic S pole
- It was discovered that charged particles experience a force when moving in a magnetic field
- It was discovered that motion of charged particles (electrical currents) create magnetic fields
  - We will learn that EVERY magnetic field arises from the motion of charged particles
  - We will also learn that there is no single separate N or S pole (like the + and – pole for E)
    - Thus N & S always appear in pairs; There is no magnetic monopole (single pole)
  - This force on a current segment in a magnetic field opens up the possibility of the motor
- Cosmic rays (charged particles) hit earth & go to poles N & S poles thus protecting the earth
  - As the earths magnetic field can go to zero and reverse, this allows for intense radiation
  - Such radiation may have induced genetic mutations on earth at those times in history
- The units of the magnetic field are the Tesla = Nt/(C m/s).
  - One Tesla is a very intense magnetic field
  - The Gauss is defined by 1 T = $10^4$ Gauss. The earth's magnetic field is about ½ Gauss.

## 27.2 Magnetic Force Equation on Charges and Currents (video)

- Magnetic Force on a moving charge is $\mathbf{F} = q\,\mathbf{v} \times \mathbf{B} = q\,v\,B \sin\theta$
  - The direction of this force is by the RHR due to the cross product
  - Examples
- Magnetic force on a current segment
  - Using calculus: $d\mathbf{F} = dq\,(d\mathbf{r}/dt) \times \mathbf{B}$ we move the dt to get $d\mathbf{F} = (dq/dt)\, d\mathbf{r} \times \mathbf{B} = I\, d\mathbf{r} \times \mathbf{B}$ or
- $\Delta\mathbf{F} = I\,\Delta\mathbf{r} \times \mathbf{B}$ giving the force on a current segment $\Delta r$ which carries a current I in a field **B**
  - This force on a current segment has the direction given by the RHR of the cross product
  - Examples

## 27.3 Magnetic Moments (video)

- Magnetic dipole moment defined: $\mu = I\,\mathbf{A}$ where I = current in a loop of area A
  - Actually all magnetic fields arise from these loops of electrical current.
  - Note that the RHR gives the direction of B due to the loop
- These dipoles are tiny magnets and thus feel a torque when in another magnetic field B:
  - Torque $\tau$ on a magnetic dipole $\mu$ in a magnetic field B is $\tau = \mu \times \mathbf{B}$
- These magnetic dipoles thus can have greater or lesser potential energy in such a field:
  - The potential energy of a magnetic dipole in a magnetic field is $U = -\mu \bullet \mathbf{B}$
  - Note that the zero of this potential energy is set when $\mu$ & B are perpendicular

## 27.4 Gauss' Law for Magnetism – One of Maxwell's 4 Equations (video)

- Gauss Law for Magnetism $\int \mathbf{B} \bullet d\sigma = 0$ = the magnetic flux through any closed surface
  - This is equivalent to :
  - There are no magnetic monopoles
  - The magnetic field has no sources and sinks (no separate N and S poles)
  - All magnetic field lines close back on themselves in closed loops

## 27.5 Motion of Charged Particles in a Magnetic Field: (video)

- Radius & Period of the path of a charged particle in a magnetic field $r = mv/qB$ $T=2\pi m/qB$
- Path of a charged particle in general is a helix around the field lines
- Note that no work can be done on a free charge moving in a magnetic field

# 28 Magnetic Field Sources

## 28.1 The Source Equation for the Magnetic Field: The Biot-Savart law: (video)

- Biot-Savart law: Magnetic fields arise from the motion of electric charge as:
  - $d\mathbf{B} = (\mu_o/4\pi)\ I\ d\mathbf{s} \times \mathbf{r}_{unit} / r^2$ where I = current, d**s** = length of wire, **dB** = mag. Field
  - $(\mu_o/4\pi) = k_m = 1E\text{-}7$ exactly thus defining the value of $\mu_o$, the permeability of free space
  - The unit vector $\mathbf{r}_{unit}$ points from the current segment $\Delta\mathbf{s}$ to the point r where **B** is located
  - Examples

## 28.2 The Magnetic Field for Simple Geometries (video)

- $B = \mu_o I /(2\pi a)$ gives the magnetic field a distance 'a' from an infinite straight wire
  - Direction using RHR
  - Compare to the Electric field due to a charged infinite straight wire: $E= \lambda/(2\pi\varepsilon_0 r)$
    - ◊ Note: The $2\pi$ values are both in the denominator
      - * but $\mu_o$ is in the reciprocal place of $\varepsilon_0$
    - ◊ Also I (current) replaces $\lambda$ (charge density) and both have a 1/r dependence –
      - * (the first power for an infinite straight line of charge or current).
- $B = \mu_o I R^2 /(2 x^2 + R^2)^{3/2}$
  - = B field on the axis a distance x from a circular loop of current I, Radius R,
  - Note directions, and dependence on distance (only valid on axis)
- $B = \mu_o n I$
  - = B field in a solenoid with $n = N / l$ (# of turns per length)
  - Note that a solenoid creates a homogeneous uniform magnetic field B
    - ◊ This is similar to what the capacitor does to create a constant E field
  - Examples

## 28.3 Force between parallel infinite conductors: - Definition of the Ampere (video)

- $F/s = \mu_o I_1 I_2 /(2\pi a)$
  - = force between two long parallel wires a distance 'a' apart with currents $I_1$ and $I_2$
  - This defines the Ampere when the force per m = 2E-7 is currents $I_1$ and $I_2$ each of 1 Amp

## 28.4 Amperes Law – One of the four fundamental Maxwell's Equations: (video)

- Ampere's law: B x distance around a closed circular loop centered on a wire: $B * Cir = \mu_o I$
- Integral calculus exact form for Ampere's law: $\int \mathbf{B} \bullet d\mathbf{s} = \mu_o I$

## 28.5 Diamagnetic Substances (like dielectric substances with E fields) (video)

- The Magnetization vector, M, = magnetic moment per unit volume and
  - Thus $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_m = \mathbf{B}_0 + \mu_o\mathbf{M} = \mu_o (H + M)$
  - For paramagnetic and diamagnetic substances, $M = \chi H$
    - ◊ where $\chi$ = the magnetic susceptibility
    - ◊ with $\mu_m = \mu_o (1 + \chi)$ substances are classified as
    - ◊ paramagnetic $\mu_m > \mu_0$ , diamagnetic $\mu_m < \mu_0$ , and ferromagnetic $\mu_m >> \mu_0$
    - ◊ Examples

## 28.6 Amperes Law as Modified by Maxwell – One of Maxwell's Four Equations (video)

- Ampere's law modified by Maxwell displacement current
  - $\int \mathbf{B} \bullet d\mathbf{s} = \mu_o I + \mu_o \varepsilon_o\, d(\int \mathbf{E} \bullet d\sigma)/dt$
  - Proof: Using a cylindrical surface around a wire ending in a capacitor then $EA = Q/\varepsilon_o$
  - then $\varepsilon_o\, d\Phi/dt = dQ/dt = I_{Maxwell}$ & use this $I_{Maxwell}$ in addition to the I in Amperes law

# 29 Faraday's Law

## 29.1 Faraday's Law for Induced Electric Fields (video)

- Faraday's discovery of induction allows the creation of voltage by moving a loop in a magnetic field
  - Either the flux can change due to the motion or orientation of the wire or loop or
  - The flux can change due to a changing magnetic field or
  - Even the motion of the source magnet can create the voltage
- Thus changing the magnetic field flux in circuit, one can induce an electric potential or voltage
  - This gives generation of electrical voltage & thus electric power from mechanical power
  - The technological leap allowed by moving energy by electricity is revolutionary
- Faraday's law of induction:
  - $V = -d\Phi_B/dt$ and $\Phi_B = \int \mathbf{B} \bullet \mathbf{d\sigma}$ the magnetic flux through an open surface (like B*A)
  - But V (induced emf) around a closed circuit is $V = \int \mathbf{E} \bullet \mathbf{ds} = -d/dt \int \mathbf{B} \bullet \mathbf{d\sigma}$
  - This last equation is the fourth of Maxwell's four fundamental equations
  - Examples

## 29.2 Lenz's Law and Motion of a Conductor in a Magnetic Field (video)

- Lenz's law states that the induced EMF will create a magnetic flux to oppose the change in magnetic flux
- EMF from the motion of a conductor in a B field:
  - $V = -B\, s\, v$ for a conductor of length s moving at v.
  - Prove this

# 30 Induction

## 30.1 Self Induction (video)

- The change in current in a wire creates a changing magnetic field on that wire and thereby creates an induced voltage which in turn opposes the voltage that creates the original current.
- Self-Inductance: the induced voltage is $V_L = - N\, d\Phi_B / dt \;=\; - L\, dI/dt$
  - ♦ The unit of inductance is the Henry (H)
  - ♦ It is difficult to compute the change in flux but it is proportional to the change in current
  - ♦ Since all electrical currents must move in closed loops, they all have self induction
  - ♦ The voltage across an inductor gives the last RCLV circuit component we need.

## 30.2 The General RLV Circuit (video)

- RLV Circuits equation $L\, di/dt + R\, I = V$
  - ♦ First order inhomogeneous linear differential equation
  - ♦ Solve using $i(t) = A\, e^{\alpha t} + B$
- : $I(t) = (V/R)\,(1-e^{-t/\tau})$ where $\tau = L/R$ is the time constant of the RL circuit
- Energy stored in the magnetic field: $U = ½\, L\, I^2$

## 30.3 Transformers – Mutual Inductance (video)

- Induction allows for the concept of a transformer which can increase or decrease AC voltage
  - ♦ The use of higher voltages means less energy loss since Power = IV = $I^2 R$
  - ♦ Note the importance of balancing voltage and current levels
    - ◊ High voltage risks arcing and electrocution,
    - ◊ High currents risk fire from overheating
- The equation for a transformer is $V_1 / N_1 = V_2 / N_2$
  - ♦ Since the transformer power input must equal power output we also have $V_1 I_1 = V_2 I_2$

# 31 Alternating Electric Currents

### 31.1 The General RCLV Circuit Equation (with constant voltage $V_0$ ) (video)

- Solve the general RCLV circuit: $L\, d^2q/dt^2 + R\, dq/dt + (1/C)\, q = V_0$
    - This is second order linear inhomogeneous differential equation
    - Use $q(t) = q_0 e^{\alpha t} + B$ Find $\alpha$ and B by substituting q(t) to get:
    - $(L \alpha^2 + R \alpha + 1/C)\, q_0 e^{\alpha t} + B/C = V_0$
    - Thus both $(L \alpha^2 + R \alpha + 1/C)$ must = 0 AND $V_0 - B/C$ must = 0
    - The second equation gives $B = CV_0$
    - Defining $\gamma = -R/2L$ $\omega_0^2 = 1/LC$ then the first equation gives $\alpha = -\gamma \pm \sqrt{\gamma^2 \pm \omega_o^2}$

### 31.2 Three cases result from the square root: (video)

- Over damped $\gamma > \omega_0$ then $q(t) = A e^{-\gamma t - \sqrt{\gamma^2 \pm \omega_o^2}\, t} + B e^{-\gamma t - \sqrt{\gamma^2 \pm \omega_o^2}\, t}$
- Critically damped $\gamma = \omega_0$ then $q(t) = A e^{-\gamma t} + B t e^{-\gamma t}$ (degenerate case)
- Underdamped $\gamma < \omega_0$ then $q(t) = A e^{-\gamma t + \omega_1 t} + B e^{-\gamma t - \omega_1 t}$ where $\omega_1^2 = \omega_0^2 - \gamma^2$

### 31.3 Discussion of Overdamped & Critically Damped

- Overdamped is a linear combination of two decreasing exponentials
- Critically damped is a linear combination of a decreasing exponential and one that is multiplied by a factor t
    - Discuss the exponential and how it overrides any finite power.

### 31.4 Discussion of Underdamped (review the harmonic oscillator general equations) (video)

# 32 Maxwell's Equations

**32.1 Lorentz force equation:** **$F = q E + q v \times B$** **( = dp /dt by Newtons equation of motion)**
(video)

**Maxwell's Equations**

**32.2 Gauss' law of electricity** $\int E \bullet d\sigma = q_{inside} / \varepsilon_0$ **or**
(video)

- $\nabla \cdot E = \rho / \varepsilon_o$ where $\rho$ is the charge density

**32.3 Gauss' law of magnetism** $\int B \bullet d\sigma = 0$ **or**
(video)

- $\nabla \cdot B = 0$

**32.4 Faraday's law of induction** $\int E \bullet ds = -d/dt \int B \bullet d\sigma$ **or**
(video)

- $\nabla \times E = -\frac{dB}{dt}$

**32.5 Ampere's law modified by Maxwell** $\int B \bullet ds = \mu_0 I + \mu_0 \varepsilon_0 d(\int E \bullet d\sigma)/dt$ **or** $\nabla \times B = \mu_o j + \mu_o \varepsilon_o \frac{dE}{dt}$
(video)
**where j is the current density**

**Gauss' & Greens Theorems:**

**32.6 The differential forms use the following two equations:**

◊ $Gauss' Theorem: \oint_S A \cdot d\sigma = \int \nabla \cdot A d^3x$ and

◊ Greens Theorem: $\int \oint_C A \cdot ds = \int (\nabla \times A) \cdot d\sigma$

# 33 Solution in a Vacuum – EM Waves

## 33.1 Overview of Maxwell's Discovery (video)

- Maxwell solved his equations in a vacuum – meaning no charges or currents and found:
- With oscillating E & B perpendicular fields at any frequency, & any amplitude with $E = cB$
- The oscillations move at exactly the speed of light, $c = (\varepsilon_0 \mu_0)^{-1\backslash 2}$ with E & B perpendicular to c
- The waves carry both energy and momenta and are transverse
- The E direction can be used to indicate the direction of polarization
- Polarization can also be circular (left or right handed) corresponding to the spin of the photon

## 33.2 Form of the EM Wave: (video)

- The wave is given by $E(x,t) = E_0 \cos(\omega t + kx + \delta)$ where $\delta$ is the phase in radians
- The angular frequency $\omega$ is the angular velocity & related to the period $T$ $(=1/f)$ by $\omega T = 2\pi$
- The wave number k is related to the wave length of a full wave by $k\,\lambda = 2\pi$
- And $E_0$ is the amplitude of the wave restricted to $E_0 = c\,B_0$
- Likewise, $B(x,t) = B_0 \cos(\omega t + kx + \delta)$ with the same values and such that $\lambda f = \omega/k = c$

## 33.3 Energy and Momentum of the EM Wave (video)

- The energy density is given generally by $u = (½)e_0\, E^2 + (1/(2\mu_0))\, B^2$
  - ♦ One must use the root mean square value for the fields as $E_{rms} = E_0/(2)^{1/2}$
    - ◊ and likewise for the B field
  - ♦ The energy and momenta are equally distributed in the E and B fields.
  - ♦ The intensity of the EM wave is the $\text{power/m}^2 = S = c\,u$ where u is the energy density

## 33.4 Doppler Effect: (video)

- Doppler effect is given by $V_{rel} << c$ by $f_o = f_s\,(1 \pm v_{rel}/c)$ ($\pm$ refers to approach or recede)

# 34 Reflection of Light & Mirrors

## 34.1 Plane Mirrors (video)

- The law of reflection is that the angle of incidence equals the angle of reflection $\theta_i=\theta_r$
- Flat Mirrors
  - The left and right handiness is reversed in a mirror (eg with handwriting)
  - A reflected image is as far behind a mirror as the object is in front and is upright
  - The image is virtual and otherwise identical to the object (except left right inversion)

## 34.2 Spherical Mirrors (video)

- Focal length is defined as the distance of an image from the mirror of an object at infinity
- The focal length of both convex and concave mirrors is given by $f = R/2$ where R is the radius
  - This can be shown using a normal to the surface
  - Note that not all rays from infinity focus exactly there but only those near the center
  - However, a parabolic mirror will focus all light at a single point.
  - Note ray tracing to form an image of an object in convex & concave mirrors (Example)
- A concave mirror gives enlarged, upright, virtual images in front of the mirror
- A convex mirror gives diminished, upright, virtual image behind the mirror

## 34.3 Image Equation for Objects and Images in General & Magnification (video)

- Let $d_o$ and $d_i$ be the distances of the object and image to the mirror then $1/d_o + 1/d_i = 1/f$
- And the magnification is $m = - d_i /d_o$
  - (if negative then image is inverted, if positive then upright)

# 35 Refraction of Light & Lenses

## 35.1 Index of Refraction & Internal Refraction (video)

- The Index of Refraction is ratio of the speed of light in vacuum to the speed in the substance
    - $n = c/v$ thus $n > 1$ always
    - Examples are diamond 2.419, Crown glass 1.523, Benzene 1.501, Water 1.333, Air 1.000293
    - Strong refraction index of lead crystal gives a prism effect with multiple colors
- Total internal reflection – critical angle–
    - $\mathrm{Sin}(\theta_c) = n_2 / n_1$
    - Water has internal reflection angle of 48.6 deg)
    - View from beneath water – how a fish sees the fisherman
- Total internal reflection used in fiber optics and prisms for binoculars
    - (glass has an internal reflection angle of 41 to 42 deg)

## 35.2 Brewster's angle and Dispersion of Light (video)

- Brewster's angle: the angle for a substance that polarizes the reflected light with $\theta_{reflect}=\theta_{refract}$
    - $\mathrm{Tan}(\theta_B) = n_2 / n_1$
- Brewster's Law: $\theta_{reflect}=\theta_{refract}$ occurs when $\tan \theta_B = n_2 / n_1$ and the reflected light is polarized
- Total internal reflection
    - Use Snell's law with $\theta_2 = 90$ deg. To get $\theta_c = \sin^{-1}(n_2 / n_1)$
- Dispersion of light:
    - Prisms – note red is least diverted (and on the pointed side of prism)
    - Rainbows: sunlight enters and is internally reflected in water drops:
        - ◊ red is bent least (rainbow top)

## 35.3 Farsightedness & Nearsightedness - Aberrations (video)

- Farsightedness (hyperopic) (use converging lens)
- Nearsightedness (myopic) (use diverging lens)
- Lenses in combination (see diagrams)
- Lens aberrations: spherical and chromatic aberration

## 35.4 Snell's Law of Refraction: (video)

- Snell's law of refraction $n_1 \sin \theta_1 = n_2 \sin \theta_2$
    - (light passing from media 1 to 2 angles rel. to normal)
- Lenses
    - Converging lens formula $1/d_o + 1/d_i = 1/f$
        - ◊ magnification $m = h_i / h_o = - d_i / d_o$
    - Sign conventions:
        - ◊ f is + for converging lens, - for diverging lense
        - ◊ $d_o$ is + if object is to the left of the lens (real object) and – if to the right (virtual object)
        - ◊ $d_i$ is + for a (real) image formed to the right of the lens by real object, and – to the left
        - ◊ m is + for an image that is upright with respect to the object, and – for inverted

## 35.5 Applications: Magnifying Glass, Telescope, Microscope (video)

- Magnifying glass magnification $m$ approx.= $(1/f - 1/d_i)\, N$ where N = dist. of near point to eye
- Telescope $m$ approx.= $-f_o/f_e$
    - where $f_o$ & $f_e$ are the focal lengths of the objective and eyepiece lens
- Microscope $m$ approx.= $-(L-f_e)N/ (f_o f_e)$
    - where L is the dist. between the lenses & N is near point

# 36 Interference & Wave Nature of Light

## 36.1 Linear Superposition (video)

- Principle of linear superposition: resultant disturbance is the sum of separate disturbances
- Interference is constructive if waves are in phase, destructive otherwise
- Thin film interference described as with gasoline on water
- Diffraction through a slit: resolving power is when the first dark band falls on the central bright band
- Diffraction grating – used to diffract light and create a spectroscope

## 36.2 Young's Double Slit and Multiple Slits: (video)

- Young's double slit experiment: $\sin\theta = m(\lambda/d)$
  - ♦ constructive with m = 0, 1, 2; destructive m = 1/2 , 3/2..
- Thin film $\lambda_{film} = \lambda_{vacuum}/n$ and
  - ♦ thus difference of distance = 2thickness + ½ $\lambda_{film}$ (due to reflection) =½ $\lambda_{film}$ , 3/2 $\lambda_{film}$…
  - ♦ then subtracting ½ $\lambda_{film}$ from each side one gets 2 t = 0, 1$\lambda_{film}$ , 2$\lambda_{film}$ , 3$\lambda_{film}$ …
  - ♦ then solving for t one gets $t = m\,\lambda_{film}/2$ where m = 0, 1, 2, 3, …

## 36.3 Single Slit Diffraction (video)

- Diffraction through a single slit gives: $\sin\theta = m\,\lambda/W$
  - ◊ where m = 1, 2, …, W=width, for destructive interference
  - ♦ $\theta_{min} = 1.22\,\lambda/D$ for the minimum resolution between two objects using an aperture D
- Diffraction grating maxima are $\sin\theta = m\,\lambda/d$ m = 1, 2, 3
  - ◊ where d is the slit separation
  - ◊ red is dispersed by the greatest angle and violet the least

# 37 Special Relativity 1905

## 37.1 Michelson –Morley Experiment c is constant! (video)

- Constancy of c, the velocity of light, to all observers presents a conflict between Newton & Maxwell
  - Maxwell EM equations predict $c = (\varepsilon_0\mu_0)^{-1/2}$ = 3E8 m/s in vacuum
    - ◊ This is true to all frames & observers
- Michelson & Morley repeatedly proved this was true using the earths motion: Explain
    - ◊ Attempts to explain c=const. with 'ether' theories etc were flawed.
- Conflict:
  - Newtonian space time is related by x'=x-Vt & t'=t thus v' = v – V ie velocities add
    - ◊ This is confirmed by our intuition and everyday experience – Examples of cars:

## 37.2 Einstein's Special Theory (video)

- Einstein assumed three postulates and allowed for a more general relationship for x & t
  - Assumption 1: The laws of physics are identical in inertially related (constant v) frames
  - Assumption 2: The speed of light in vacuum is a constant.
  - Assumption 3: The relationship between x & t in two frames is linear for the 4 dimensions
- Einstein showed that space (length) and time are not each invariant but transform as a 4 dim. vector
  - This 4-vector of space-time described an event for one observer & related it to another observer

## 37.3 Lorentz Contraction & Time Dilation (video)

- Lorentz Contraction: One can then show that length is contracted by $L = L_0\,(1-v^2/c^2)^{1/2}$
  - where L is the observed length and $L_0$ is the length in its own rest frame
- Time Dilation: One can also show that time is expanded by $t = t_0\,/(1-v^2/c^2)^{1/2}$
  - where t is the observed length and $t_0$ is the length in its own rest frame
  - These effects are only about 1% when one gets to a tenth of the speed of light: v/c =1/10
  - Below that relativity is essentially negligible. Yet effects explode near v=c.

## 37.4 Relativistic Energy Equation (video)

- The old formula for $KE = \mathbf{p}^2/(2m)$ is now replaced by:
    - ◊ $(E/c)^2 - P_x^2 - P_y^2 - P_z^2 = m^2c^2 = E^2/c^2 - \mathbf{P}^2$
  - Now using $E^2/c^2 - \mathbf{P}^2 = m^2c^2$ to solve for E we get
  - $E = \pm\sqrt{c^2p^2 + m^2c^4}$ *and when* $p = 0,\ E = \pm mc^2$ , the famous Einstein equation
- In relativity neither mass nor energy is separately conserved
  - but only their combination via $E=mc^2$
- The negative sign was ignored for 20 years until it was shown to correspond to 'antimatter'
  - Antimatter is identical to matter except of opposite charge
    - ◊ It annihilates corresponding matter of the same type into pure light.
- Next we solve $E^2/c^2 - \mathbf{P}^2 = m^2c^2$ for m (choose units with c=1):
  - $m = \pm\sqrt{E^2 - p^2}$ giving 3 cases:
  - E>p giving m >0 and v<c This is ordinary matter and must move slower than c
  - E=p giving m = 0 and v=c These massless particles, such as photons, always have v=c
  - E<p giving m imaginary and thus v>c are called tachyons and must move faster than light
- Physicists have wondered about m<0 and if it would give antigravitiy
  - But no m< 0 has been found

## 37.5 Lorentz Transformation (video)

- The Lorentz transformation derived: x' = L x where $x = (ct, x, y, z) = (x^0, x^1, x^2, x^3) = x^\mu$
  - This set of four 'coordinates' of an event, is a 4 dimensional vector under L
  - A sphere of light, ct=r must be seen the same by all observers thus $c^2t^2-r^2$ = invariant
  - Compute this in two dimensions to get $(x'^0, x'^1) = (L^0{}_0, L^0{}_1, / L^1{}_0, L^1{}_1)\,(x^0, x^1)$ then
  - One obtains $(L^0{}_0, L^0{}_1, / L^1{}_0, L^1{}_1) = (\text{ch}\varphi, \text{sh}\varphi\ /\ \text{sh}\varphi\ , \text{ch}\varphi\ )$ where $\text{th}\ \varphi = v/c$
    - ◊ because of $\text{ch}^2\varphi - \text{sh}^2\varphi = 1$ (compare to $\cos^2\theta + \sin^2\theta = 1$)

◊ $$ch\phi = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, sh\phi = \frac{v/c}{\sqrt{1-\frac{v^2}{c^2}}}$$

### 37.6 The Relativistic Scalar Product in 4 Dimensions (video)

- The scalar product, defining the metric properties of the space is $A \cdot B = g_{\mu\nu} A^\mu B^\nu$ where
- The metric for this invariant is $g_{\mu\nu}$ is defined by $g_{\mu\mu} = (+1, -1,-1,-1)$ and $g_{\mu\nu} = 0$ off diagonal
- Thus $d\tau^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$ is invariant and is called the proper time: $d\tau^2 = c^2\, dt^2 - d\mathbf{r}^2$
  - ◊ because it gives the invariant time interval on a clock on the particle that is moving
  - ♦ As time is part of a 4 vector, we cannot effectively use it to take derivatives
    - ◊ One must use $d\tau$ thus giving a 4-vector velocity of $v^\mu = c\, dx^\mu / d\tau$
      - * (note that 'c' give it dimensions of vel)
    - ◊ and one can verify that the invariant length of this vector is always c : $g_{\mu\nu} v^\mu v^\nu = c^2$

### 37.7 Four Momentum Vector (video)

- The 4-vector momentum is thus defined as mass times velocity:
  - ♦ $p^\mu = m\, v^\mu$ then $g_{\mu\nu} p^\mu p^\nu = m^2 c^2$
  - ♦ Thus energy & momentum form a 4 vector: $(E/c, P_x, P_y, P_z) = P^\mu$ and transforms like $dx^\mu$
- When $g_{\mu\nu} p^\mu p^\nu = m^2c^2$ is written out it becomes: $(E/c)^2 - P_x{}^2 - P_y{}^2 - P_z{}^2 = m^2c^2 = E^2/c^2 - \mathbf{P}^2$
  - ♦ This is the relativistic equation relating energy, momentum and mass
    - ◊ It replaces $E = p^2/(2m)$ which was valid in Neutonian mechanics

# 38 General Relativity & Astrophysics 1916

## 38.1 Foundational Need for General Relativity (video)

- Special relativity addresses observers moving with relative constant velocity only
- General relativity deals with cases where one observer is accelerated relative to the other
- Rotating platform: Einstein argued that a rotating platform gives a non-Euclidian (curved) geometry
  - ♦ With increasing r, the Lorentz contraction shortens circumferences to smaller values
    - ◊ Also as one moves outward, clocks slow down because of time dilation
  - ♦ Far from the center, where v is almost equal to c,
    - ◊ the circumference is near 0 & time stands still
  - ♦ So space and time in accelerated frames is unquestionably curved (not 'flat')

## 38.2 Elevator experiment: (video)

- Einstein compared an accelerated elevator to the same one in gravity with a=g
  - ♦ No experiment with regular matter would distinguish g from a as all mass has the same g
  - ♦ Yet light is not bent by gravity (as per Newton) but light 'appears' bent with acceleration
  - ♦ Einstein argued that by symmetry, light should be bent the same amount by g as by a
    - ◊ This violates the Newton formula for gravity as light has a mass of zero
    - ◊ His prediction that light from a distant star is bent by the sun was verified
- Gravity (and acceleration) is thus seen as a warped space time
  - ♦ Masses follow paths which are geodesics
- The integration of Einstein's theory is still not reconciled with modern theories of other forces

## 38.3 Rotating Platform Mathematically: (video part 1, part 2)

- A rotating platform circumference is shortened by the Lorentz contraction: $C = C_0\,(1-v^2/c^2)^{1/2}$
  - ♦ One can compute at what point the circumference begins to get smaller and at v=c is zero
- At larger distances from the center, time dilation effects slow time by $t = t_0\,/(1-v^2/c^2)^{1/2}$
  - ♦ where t is the observed length and $t_0$ is the length in its own rest frame
- In both equations, $v = r\,\omega$ where $\omega$ is the angular velocity of the platform

## 38.4 The Mathematical Theory of General Relativity: (video)

- The mathematical theory of curved spaces is called Riemannian or differential geometry
- The fundamental concept is the metric $g_{\mu\nu}$ which is used to define scalar products
  - ♦ Length & angle are defined from the scalar product
- Particles (as well as light) follow the shortest distances (called geodesics) in curved spaces
- Einstein's equation relates $g_{\mu\nu}$ for 4- space to $T^{\mu\nu}$ , energy-momentum tensor density

# 39 Foundations of Quantum Mechanics – Particles & Waves

## 39.1 Cavity Radiation - Plank (video)

- Cavity radiation refers to EM radiation from a hole inside a substance
  - ◊ -also called blackbody radiation
  - ♦ Is dependent upon the temperature and independent of the substance making the cavity
- Cavity radiation was found to have wavelength spectra that could not be explained by theory
- Plank (1900) proposed that the walls consist of oscillators
  - ♦ Furthermore he proposed that these oscillators can emit & absorb only certain quanta
  - ♦ Specifically: $E_{em} = n\,h\,f$
    - ◊ where n = 1,2,.. f = the frequency of radiation, and h is a constant 6.6260755E-34

## 39.2 Photoelectric Effect: (video)

- Photoelectric effect is the emission of electrons from a metal when radiated by ultraviolet light
- The following problems emerged in understanding the experimental results:
  - ♦ 1: The energy of the electrons is independent of the light intensity but depends only on f
  - ♦ 2: Below a given f of light, no electrons are emitted no matter how intense the light is
  - ♦ 3: The effect of emission is immediate no matter how low the intensity
  - ♦ These problems were counter to the Maxwell theory of EM radiation
  - ♦ The results of cavity radiation was also counter to the Maxwell theory.
- Einstein explained both phenomenon and founded quantum theory postulating photons
  - ♦ Where: $E_{em} = hf$
  - ♦ Thus light consisted of these 'quanta' of pure massless energy also with momenta $P=h/\lambda$
  - ♦ Thus the view of EM radiation as oscillating E and B fields is an approximation to photons

## 39.3 Compton Scattering of photons and electrons: (video)

- Arthur Compton in 1923 scattered photons from electrons
  - ♦ and showed that $\lambda'-\lambda = (h/mc)(1-\cos\theta)$
  - ♦ This confirmed the Einstein photon hypothesis experimentally

## 39.4 De Broglie Wave Hypothesis: (video)

- Louis De Broglie in 1923 proposed the same photon equations $E_{em} = hf$, $p=h/\lambda$ apply to matter
  - ♦ Thus given a particles energy E and momentum p, one can compute an associated f & $\lambda$
  - ♦ Matter has a short wave length and thus we do not normally 'observe' the wave nature

## 39.5 Davisson – Germer Experiment (video)

- In 1927, Davisson & Germer & Thompson confirmed wave interference effects scattering $e^-$
  - ♦ This scattering of $e^-$ from a crystal gave interference patterns
    - ◊ These were only possible for a wave like X rays
  - ♦ This experiment confirmed the De Broglie hypothesis that matter was also a wave.

## 39.6 The Wave Equation for Matter (must replace the old Newton equation for particles) (video)

- In 1925, Erwin Schrödinger proposed his equation for the 'motion' of this 'matter wave' $\Psi(x,y,z,t)$
- In 1925 Werner Heisenberg also proposed an alternate formulation for $\Psi$ in terms of matrix theory
- In 1926 P.A.M. Dirac presented a unifying mathematical theory that showed these theories equivalent

## 39.7 Heisenberg Uncertainty Principle (video)

- Heisenberg later showed that $\Psi$ contains information on both the particles position and momenta BUT
  - ♦ to know more about the position one looses knowledge of the momenta and conversely:
  - ♦ Heisenberg uncertainty principle gives the product of these uncertainties: $\Delta x\, \Delta p >= h/4\pi$
  - ♦ Also one has an equivalent equation for energy and time: $\Delta t\, \Delta E >= h/4\pi$
  - ♦ Heisenberg's uncertainty principle has deep implications for what is simultaneously knowable

### 39.8 Particle in a Box (video)

- A particle of mass m, in a box of length L must have an integer number of half waves
  - Thus $n \lambda / 2 = L$ thus $\lambda = 2L / n$
    - thus $p_n = h/\lambda = n h / 2L$ resulting in a discrete set of momenta
  - Using $E = P^2/(2m)$ we get $E_n = n^2 h^2/(8m L^2)$
    - giving the discrete energies of a particle in a box
    - In particular it states that the lowest energy is not zero.

# 40 Atomic Theory

## 40.1 The Model of the Atom: Prior to 1911 and afterward: (video)

- The Thompson model of the atom held that positive charge was spread out like a pudding.
- In 1911 Rutherford scattered $\alpha$ particles from gold foil and obtained large deflections.
  - This implied the positive charge was heavy and highly concentrated – not spread out.
  - This showed the nuclear size was ≈ 1E-15m called a Fermi or a femtometer.
- This raised the problem of why the electron did not spiral into the center with infinite radiation
- Atomic spectra was observed at discrete frequencies rather than continuous emissions
  - This implied discrete orbits for the electron but what equations would make this work?

## 40.2 Bohr's Model of the Atom: (video)

- In 1913 Bohr proposed his model of the atom with quantized orbits and discrete transitions
- The Bohr model assumes that angular momentum is quantized. $L_n = n\, h/(2\pi)$,
  - But there is no explanation for why this is true.

## 40.3 Pauli Exclusion Principle: (video)

- The Pauli exclusion principle prevents two electrons from being in the same shell simultaneously
- The same principle is valid if the particles are identical and have spin of ½ 3/2 etc h/2Pi
- But it does not apply to interger spin particles of spin 0, 1, 2,…

## 40.4 Stimulated Emission of Radiation: (video)

- Einstein predicted that if an excited atom is hit with a photon of the decaying energy then ..
  - rather than being absorbed, the photon will stimulate the emission of another photon in phase
  - This principle is the basis for the operation of a laser
  - LASER means Light Amplification by Stimulated Emission of Radiation
- X Rays were discovered by Wilhelm Roentgen by hitting electrons on a metal target

## 40.5 Atomic Spectra Equation: (video)

- Atomic spectra was observed to obey: $1/\lambda = R(1/n_1^2 - 1/n_2^2)$ with terminology of:
  - $n_1 = 1$ Lyman series , $n_1 = 2$, Balmer series, $n_1 = 3$ Paschen series …
  - Bohr's model of quantized orbits assumed a quantized angular momentum of
    - ◊ $L_n = n\, h/(2\pi)$, n= 1,2
    - ◊ This assumption in addition to the classical equations gave workable orbits:
    - ◊ One balances Coulomb force with centripetal force: $mv^2/r = kZe^2/r^2$ Z=# protons
    - ◊ Using these two equations, the radius is $r_n = h^2\, n^2 / (4\pi^2 kme^2 Z) = 5.29\text{E-}11\ n^2/Z$
    - ◊ The electron's energy is KE+PE = E = $(1/2)\, mv^2 - kZe^2/r$
    - ◊ Thus $E_n = 2\pi^2 mk^2 e^4/h^2)(Z^2/n^2) = -13.6\text{ eV } Z^2/n^2 = -2.18\text{E-}18\text{ J } Z^2/n^2$
      - * Note that the factor 13.6 eV is the ionization energy of hydrogen (Z=1 & n=1)
    - ◊ Since $1/\lambda = f/c = E/hc$ then $1/\lambda = 2\pi^2 mk^2 e^4/(ch^3)\ (Z^2/n^2)$ explaining the $1/\lambda$ law.

## 40.6 Explanation of the Bohr Hypothesis by De Broglie: (video)

- De Broglie: If the electron 'wave' had to meet constructively with itself then
  - ◊ Cir. $= 2\pi r = n\, \lambda = n\, h/p$
  - ◊ Consequently we get quantized angular momentum as $r\, p = L = n\, (h/ 2\pi)$

## 40.7 Schrödinger Equation: (video)

- The Schrödinger equation solution to the hydrogen atom gives the following energy levels:
  - The principle quantum number, n = 1, 2, 3, …..
    - ◊ The principle quantum numbers 1, 2, 3,..are denoted by the shell names: K, L, M
  - The orbital angular momentum $l$ has the values 0, 1, 2, 3, … (n-1)
    - ◊ where $L = ((l(l+1))^{1/2})h/2\pi$
  - The orbital angular quantum numbers 0, 1, 2, ..are denoted by the letters s, p, d, f, g, h,
  - There is also a 'magnetic quantum number' that has the values $-l, -l+1, \ldots l-1, l$
    - ◊ The magnetic quantum number was seen when levels were split with a magnetic field
    - ◊ It is known to correspond to the z component of the angular momentum $L_z$
  - A final splitting of the energy levels occurred due to the z component of the spin of the electron

- The associated counting of levels now exactly counts for the number of electrons in each orbit
  - ◊ The maximum number of electrons in a shell are $2(2\,l+1)$
  - ◊ The denotation of electrons in a shell is say: $2p^5$ thus $n=2$, $l=1$, and with 5 electrons
  - ◊ Thus the configuration of Carbon (6 electrons) is $1s^2\ 2s^2\ 2p^2$

## 40.8 Pauli Exclusion Principle: (video)

- Pauli Exclusion Principle:
  - No two identical fermions can occupy the same state at the same time
  - A Fermion is an elementary particle with a spin of ½, 3/2, 5/2, 7/2, … times $h/(2\pi)$
    - ◊ Electrons, protons, neutrons, neutrinos, muons, … are all Fermions
  - A Boson is an elementary particle with a spin of 0, 1, 2, 3, … times $h/(2\pi)$
    - ◊ e.g. a photon, pion, kaon, …
    - ◊ Bosons actually ‘prefer’ to be in the same state rather than being prevented
- Without the exclusion principle, all electrons would go to the lowest state & not fill shells
  - ◊ Then without a tendency to fill shells,
    - ∗ there would be no chemical bonding, & no biology & no life!
  - ◊

# 41 Nuclear Theory & Radioactivity

## 41.1 Nucleons: (video)

- Nucleons are protons or neutrons – the particles that make up the nucleus of the atom
  - ♦ The neutron was discovered in 1932 by Chadwick
    - ◊ The neutron has a mass slightly larger than the proton
- The atomic number, Z =the number of protons; A the mass number = the number of nucleons
- Designation of the nucleus:
  - ♦ A nucleus is written as ${}^{A}_{Z}X$ where X is the chemical element corresponding to Z
  - ♦ Isotopes are nuclei with the same number of protons but differing numbers of neutrons
  - ♦ The nuclear forces felt by both the p and n are essentially identical
  - ♦ The binding energy is the amount of energy needed to separate the nucleons
  - ♦ The mass defect is the binding energy expressed in mass equivalence via $E = mc^2$
  - ♦ The binding energy per nucleon is greatest in mid-range of A (Fe) and less in Li and U
- The approximate radius of the nucleus is $r = 1.2E\text{-}15\ A^{1/3}$

## 41.2 Nuclear reactions: (video)

- ♦ Rutherford (1919) observed the first 'transmutation of an element' with $\alpha$ + N -> O + H
- ♦ Radioactivity is the decay or disintegration of an unstable nucleus
- ♦ $\alpha$ decay:
  - ◊ The emission of an alpha particle or He nucleus (2p+2n) – easy to stop
    - * Example of $\alpha$ decay ${}^{238}_{92}U$ -> ${}^{234}_{90}Th$ + ${}^{4}_{2}He$ + 4.3 MeV of energy
- ♦ $\beta$ decay:
  - ◊ The emission of an electron (or positron) via n -> p + $e^-$ + $\nu_e$ - not hard to stop
    - * Example of $\beta$ decay ${}^{234}_{90}Th$ -> ${}^{234}_{91}Pa$ + ${}^{0}_{-1}e + \nu_e$
  - ◊
- ♦ $\gamma$ decay:
  - ◊ The emission of a high energy photon releasing energy – needs lead to stop
- ♦ n decay:
  - ◊ The emission of a neutron directly from the nucleus

## 41.3 Half-Life & Radioactivity: (video)

- ♦ Half-life is the time required for half of a substance to undergo disintegration
- Radioactive disintegration obeys $N = N_0 e^{-\lambda t}$ thus $N/N_0 = 1/2 = e^{-\lambda T1/2}$
  - ♦ Radioactive decay obeys: $dN(t) = -\lambda N_0\, dt$ with the solution: $N = N_0 e^{-\lambda t}$
- Taking In of both sides we get In ½ = $-\lambda T_{1/2}$ thus $T_{1/2} = \ln 2/\lambda$ thus relating $\lambda$ to $T_{1/2}$
  - ♦ Radioactive dating: Carbon 14 has a half life of 5730 years
- The Becquerel (Bq) is the unit of radioactivity = 1 disintegration per sec
  - ♦ The Currie (Ci) is another unit of activity: 1 Ci = 3.70E10 Bq = 1 gr of pure radium

## 41.4 Biological Effects of Radiation (video)

- Ionizing radiation (charged particles or $\gamma$) knocks electrons from atoms & damages cells
  - ◊ The SI unit of ionizing radiation is the Coulomb per kg or C/kg
  - ◊ The Roentgen (R) = 2.58E-4 C/kg is a more common historical unit
- ♦ Yet this measures only the ionization effect and not the effect on tissue for which we use:
  - ◊ Absorbed Dose = (Energy absorbed) / (Mass absorbing) unit = Grey (Gy)=J/kg
  - ◊ Radiation Absorbed Dose (RAD) = 0.01 Gy is another common unit
- ♦ To compare the damage of absorbing different kinds of radiation we define:
  - ◊ Relative Biological Effectiveness (RBE)=(Dose of 200KeV X-rays Effect) /(Dose )
  - ◊ Then Biologically Equivalent Dose (rems) = Absorbed Dose (in rads) x RBE
    - * rem stands for roentgen equivalent man
  - ◊ Humans receive an average dose of 360 mrem/yr from all sources
    - * (cosmic rays 28, earth 28, internal 39, Radon 200, Medical/dental 43,..
    - * The general population should not get more than 500 mrem / yr
    - * Workers should not get more than 5 rem / year (eg dental assistant)

## 41.5 Nuclear Fission and Fusion (video)

- Nuclear fission: ${}_{0}^{1}n + {}_{92}^{235}U \rightarrow {}_{92}^{236}U \rightarrow {}_{56}^{141}Ba + {}_{36}^{92}Kr + 3\,{}_{0}^{1}n$
  - when heavy nuclei are split into two more stable nuclei with energy release
- Nuclear fusion: ${}_{1}^{2}H + {}_{1}^{3}H \rightarrow {}_{2}^{4}He + {}_{0}^{1}n$
  - when light nuclei are combined at temperatures in the sun to make heaver ones
- Nuclei can be plotted in two dimensions on an A vs Z plot or an N vs Z plot showing all nuclei
  - Either plot shows every possible nucleus and is very effective in visualizing decays
-

# 42 Elementary Particle Theory

## 42.1 Elementary Particles: (video)

- Elementary Particles:
    - are classified into categories, based upon spin value, interaction strength…:
- Spin: Fermions have half integer spins (½, 3/2, 5/2 ..) $\hbar$ , Bosons integer spins (0,1,2..) $\hbar$
- Strongly interacting particles are called Hadrons (participate in the nuclear or strong force)
    - Hadrons that are Fermions are called Baryons e.g. p, n, $\Sigma$, $\Lambda$, $\Xi$, $\Omega$ …
    - Hadrons that are Bosons are called Mesons e.g. $\pi$, K, $\eta$, ...
- Leptons (6) are Fermions that are not Hadrons (have no strong interactions) eg e, $\mu$, $\tau$, $\nu_e$, $\nu_\mu$, $\nu_\tau$,

## 42.2 The Standard Model of Elementary Particles: (video)

- Quarks (6): are the more fundamental particles that compose all the Hadrons: u, d, s, c, b, t
- Gauge particles intermediate the forces: Gravity graviton, EM $\gamma$, Weak Z, W , Strong gluon
- Particles can be specified in classes by their quantum numbers
    - ◊ (charge, strangeness, isospin, …)
- Particles plotted in these quantum number spaces have patterns representing ‘groups’
- These group theory patterns have given a basic order to the more than 300 particles
- The model for this group theory is called the standard model with the following general idea:
    - All hadrons are composites made of quarks (eg p = (d+u+u), n = (d+d+u), $\pi^-$=(d+anti u)
    - The 6 leptons and 6 quarks have very parallel interactions for EM and Weak interactions

# 43 Cosmology (video)

- Cosmology is the study of the structure and evolution of the universe
  - Hubble discovered that distant galaxies are all moving away from each other
  - Thus the universe is expanding, and furthermore this expansion is accelerating
  - The expansion should slow due to gravity but dark energy is causing the increase
  - The big bang is estimated to have occurred about 13.7E9 years ago
  - The cosmic background radiation is today at a temperature of about 2.7 K
  - There are approximately 1E11 stars in our galaxy (the Milky Way)
    - There are approximately 1E11 galaxies in our universe
  - Hubble's law of expansion: $v = H\ d$ where H is the Hubble parameter 0.022 m/(s ly)
- All of matter that is known to scientists constitutes only 5% of the known substance of the universe. The rest is dark matter and dark energy. We have no idea what these are!

# 44 Appendix - Mathematics Background

1. **The Number System:** (video)
    a. The Finite Numbers:
        i. Finite Numbers:
        ii. Originate in the acts of counting and measuring then arithmetic operations:
        iii. The number system operations: + - * / ^
        iv. Integers
        v. Positive integers / whole numbers (counting) 1, 2, 3,… with + - * / $a^b$ = a^b
        vi. Negative integers (inverse addition) -1, -2, -3… (from inverse addition) 3 + x =0 or x = -3
        vii. Zero – for a long time this was not a number, It was not apparent that a symbol for nothing was needed
        viii. Rational numbers / fractions = a/b (ratios of integers from inverse multiplication) a * x =1 or x=1/a
        ix. Irrational numbers / non-repeating decimals (from inverse exponentiation) $a^b$ such as $(2)^{1/2}$, also □□□e, □
        x. Complex numbers (also from inverse exponentiation with negative numbers) $(-1)^{1/2}$ = i
        xi. imaginary numbers and complex values = a + ib
        xii. With infinity, the complex numbers close under all operations.
        xiii. Unit circle / complex numbers: $e^{ix}$□ = cos x□□ + i sin x□ □□ also z = u + iv = $re^{i\theta}$□ = r cos θ□□ + i r sin θ
2. **Infinite Numbers:** (video)
        i. Cantor – concept of 1 to 1 matching – multiple levels of infinity
            1. Infinity of counting 1,2,3,… Note same value as even integers
            2. Same as the infinity of rational numbers a/b
            3. Infinity of real numbers
            4. Infinity of functions
3. **'Scientific notation & Numbers to Other Bases:** (video)
    a. Scientific Notation:
        i. 1.23456E3 = $1.23456*10^3$ = 1234.56 likewise 4.56E-2 = 0.0456
    b. Binary numbers & Other Bases:
        i. 10111.0011 or even in scientific notation as 1.1001E101
        ii. Other number bases are often taken as 8 or 16 symbols.
4. **Numerical Uncertainty & Order of Magnitude Numbers** (video)
    a. Numerical Uncertainty
        i. 1.23 = 1.23???...
        ii. Addition & Subtraction – truncate with alignment of least accurate value
        iii. Multiplication & Division – retain only the least number of significant digits :
    b. 'Order of magnitude numbers'
        i. 2E32 or maybe just 1E32 and calculations. Problems:

5. **Data & Metadata:** (video)
    a) Data is meaningless by itself except as an abstract number.
    b) We generally need a form like < data | units | metadata > where metadata contains the description.
        i) For example < 68.3| > is simply a numerical value without metadata for the values meaning
        ii) While < | kg | Jack's mass > is metadata without a value
        iii) Then <68. 3|kg |Jacks mass > is both metadata (including units) and the data.
    c) Data usually takes the form of a scientific number but can also be symbolic such as e, π, i, ∞

6. **Supporting concepts in Logic – Origin in the special operations of logical & rational thought:** (video)
    a) Special notations with examples:
        i) There exists ∃   $\exists\ x \in \{R\}\ \ni x = x^2 \Rightarrow x = 0$ or 1 or ∞
        ii) Therefore ∴   I think ∴ I am
        iii) Member of ∈
        iv) Such that ∋
        v) Implies ⇒
        vi) For all ∀
        vii) Isomorphic 1-1

viii) Infinity $\infty$
ix) Equality = and not equal $\neq$
x) Equal by definition or identical to $\equiv$
xi) Greater than >, less than < and also greater than or equal to >=
xii) Includes $\subset$

b) Logic & Set Theory
i) Elements 1, 0 or T, F
ii) Operations AND, OR, NOT, NOR NAND, EQV, (16 operations)
iii) And $\vee$
iv) Or $\wedge$
v) Not $\neg$
vi) Union $\bigcup$
vii) Intersection $\bigcap$
viii) Set {s}
ix) Null Set $\varnothing$

**7. Basic Algebra – Origin in expressing relationships among quantities represented by symbols.** (video)

c) Fundamental Operations:
i) Generally we then take the relationships and derive simpler equivalent relationships
ii) Equations: Solve by doing the same thing to both sides of an equation
iii) Powers add $x^a * x^b = x^{(a+b)}$ $(x^a)^b = x^{(a*b)}$
iv) Factoring $x^2 - y^2 = (x+y)*(x-y)$
v) Quadratic Equation solutions $ax^2 +bx +c =0$ solution: $x = (-b \pm \sqrt{b^2-4ac})/2a$
vi) Linear equations: y = mx+b gives b as intersection at y=b for x=0 and with m= the slope
vii) Simultaneous equations - solution is intersection

d) Logarithms log a + log b = log (a*b) and log a - log b = log (a/b)
i) $y = \log_a x$ implies $x = a^y$
ii) $b \log_a(x) = \log_a(x^b)$
iii) $\log_a b = \log_e b / \log_e a$ this allows one to convert log from one base to another

e) Socioeconomic variables (population, electric use)
i) Are generally exponential in time and thus their logarithms are linear in time
ii) Ratios of socioeconomic variables are relatively constant
iii) Income and net worth are generally log normal (their logarithms are a normal distribution)

**8. Geometry – Origin in characterizing geometrical shapes in 2 and 3 dimensions** (video)

a) Angular degrees & radians $\theta =$ s/r
b) Area & volume
(1) Rectangle & rectangular solids, parallelogram area
(2) Triangle A = ½ base * height
(3) Circle $C=2\pi r$ $A= \pi r^2$ Sphere $A = 4 \pi r^2$ $V = (4/3) \pi r^3$
(4) Cylinder $\pi r^2$ * height

**9. Trigonometry –**

**a) Origin is in the ratios of sides of similar triangles (which have identical angles)** (video)
i) Right triangles are the most fundamental shapes and all other (non-curved) can be made from these
ii) Basic triangle x y r: $\sin\theta = y/r$ , $\cos\theta = x/r$ , $\tan\theta = y/x = \sin\theta / \cos\theta$
iii) The problem is then to relate these ratios (say for r = 1) to $\theta$ as a fraction of a circle (or better yet in radians)
iv) $\sin^2\theta + \cos^2\theta = 1$ review trig identities

**10. Series expansions – Originate in solutions to equations for transcendental values** (video)

- $e^x = 1 + x + x^2/2! + x^3/3! + x^4/4!$ …..
- $\log(1+x) = x - x^2/2 + x^3/3 -$
- $\sin\theta = \theta - \theta^3/3! + \theta^5/5!$ and $\cos\theta = 1 - \theta^2/2! + \theta^4/4!$
- or $\sin x = (e^{ix} - e^{-ix})/2i$ $\cos x = (e^{ix} + e^{-ix})/2$ and $\cos^2 x + \sin^2 x = 1$
- $sh(x) = \sinh(x) = (e^x - e^{-x})/2$ $ch(x) = \cosh(x) = (e^x + e^{-x})/2$ give the hyperbolic functions $ch^2x - sh^2x = 1$

- Binomial series $(a + b)^n = a^n + n\, a^{(n-1)}b + n(n-1)\, a^{(n-2)}\, b^2/2! +$ (note divide by the larger of a or b to make b small
- Taylor series $f(x) = f|_{(x0)} + f'|_{(x0)}\, (x-x_0) + (1/2!)\, f''|_{(x0)}\, (x-x_0)^2$ …

## 11. Scalars, Vectors, Matrices, Tensors Linear Algebra & Matrix Theory (video)

- Scalar: Specified by a single real number: time, temperature, mass, volume, energy
- Vector: An ordered n-tupe of real numbers: (x, y, z) or $(x_1, x_2, x_3)$ eg (1,-5,0)
  - The dimensionality of a space is the number of numbers needed to specify a point.
  - A vector in that space has exactly that many ordered numbers in its specification
  - Examples are position, velocity, acceleration, force, momentum
  - The components of a vector must transform exactly like the coordinates under a transformation.
- Matrix: A two dimensional array of numbers $C_{ij}$ where i is the row and j is the column
  - A matrix is often used to perform a linear transformation on a vector
  - Also used to solve a set of simultaneous linear equations. <example of rotations>
  - Commutation of matrices – a matrix as a linear operator [A,B] = AB – BA
- A scalar is a tensor of rank 0, a vector is a tensor of rank 1, a matrix is a tensor of rank 2
- Operations with vectors:
  - Graphical (as used in high school)
  - i, j, k unit vectors as used in some engineering texts (do not use this notation)
  - r = (x, y, z) or $(x_1, x_2, x_3)$ or simply as $x_i$ or for example (3, -2, 5)
  - Linear Vector Space (LVS): Addition, subtraction, & multiplication by a scalar <examples>
  - Metric Space (LVS with a scalar product): Scalar product $\mathbf{A} \cdot \mathbf{B} = |A|\,|B| \cos\theta$
    - Note that this contains the Pythagorean theorem
    - Thus $\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z = g_{\mu\nu} A^\mu B^\nu$ where generally the metric can be functions of x

## 12. More Advanced foundations of vector notation for LVS (Linear Vector Spaces) (video)

- Vectors are denoted | a, b, c …> for the space and < a, b, c …| for the dual space.
  - A finite or infinite dimensional LVS is spanned by an orthonormal set | i > where i ranges over indices
  - The scalar product is then given by $\langle i \mid j \rangle = \delta_{ij}$ or perhaps with a metric g as $\langle i |g| j \rangle$
  - The decomposition of unity is given by $1 = \Sigma\, | i \rangle \langle i |$ where $\Sigma$ represents a sum or integral over i
- Abstract operations such as $| x \rangle = L\, | y \rangle$ can be put into a given basis by the decomposition of unity as
  - $\langle j | x \rangle = \langle j | L\ \Sigma\, | i \rangle \langle i |\ | y \rangle$ which gives the familiar form: $x_j = \Sigma_i\ L_{ji}\, y_i$
- A LVS with a commutator defined is a Lie Algebra: $[L_i, L_j] = c_{ijk}\, L_k$
  - (where c is antisymmetric and obeys the Jacobi identity)
  - A Lie algebra generates the transformations of a Lie (continuous group) via $G(s) = e^{sL}$ (s = real #)

## 44.1 Energy & Power

Some Useful Values - see: http://www.eia.doe.gov/mer/ for the latest values

### 1. Energy Units

- 1 Joule = F*d= Newton * Meter
- 1 Calorie = 4186 J = heat necessary to raise the temp of 1 Kg of H2O by 1 deg C
- 1 calorie = heat necessary to raise the temp of 1 gram of H20 by 1 deg C
- 1 BTU = 1055 J = heat necessary to raise the temp of 1 lb H20 by 1 deg F
- 1 KWHR = 3.6 E6 J = 1 Kilo Watt of power times 1 hour
- 1 Therm = 1E5 BTU = heat content of 100 ft3 of natural gas
- 1 Kilo Ton = 1E12 cal = energy in one thousand tons of TNT
- 1 Barrel Oil = 5.6E6 BTU = energy of crude oil per barrel
- 1 KG chemical fuel = 1E7–5E7 J energy range of chemical processes (bread 1100KC/lb to Nat Gas)
- 1 KG nuclear fuel = 1E14 = fusion or fission process
- 1 KG matter-antimatter = 1E17 = total matter antimatter annihilation
- 1 kitchen match = 1 BTU
- 1 ev = 1.6E-19 J = energy from 1 electron falling between 1 Volt potential difference

### 2. Power Units

- 1 Watt = 1 J / s
- 1 HP = 745.7 W = power from 1 horse
- 1 person's energy per day = 2000 C/day = about 100W
- 1 person's maximum power = 100 W continuous, 400 W peak
- Solar power per area
  - 1.4 KW / $m^2$ = Max value above atmosphere
  - 1KW / $m^2$ = Max at equator at noon on a clear day
  - 200 W / $m^2$ = US & SC year round Average (day, night, rain sun)
  - 1.7E17 W / whole earth= total solar power to the earth
- Wood gives 2 tons/acre / year = 0.2 W /$m^2$ thus is 1% efficient < error (either 2 W/$m^2$ or 0.1%)
- Average US person total energy very approximately is 18 KW / person
- Person walking uses 260 BTU/mile
- Person on bike uses 80 BTU/mile
- Automobile uses 10,000 BTU/mile if 1 occupant only

### 3. Efficiency (approximate values)

- Agriculture
  - Primitive use is 0.2 to 0.5 C to get 1 C
  - Modern use is 15 C to get 1 C
- Heat Pump (eg in SC) 200%
- Oil or NG furnace 85%
- Passive Solar 45%
- Active Solar Cells to Elec. 10%
- Incandescent Light 3%
- Fluorescent light 15%

### 4. Population

- US 300 Million (October 2007)
- The Earth 6 Billion (2006)
- State of SC 4 Million
- Doubling Time: % * t = 72

### 5. US Energy Usage:

| | 2007 | 1973 |
|---|---|---|
| Residential | 21% | 19% |
| Commercial | 18% | 15% |
| Transportation | 29% | 26% |
| Industrial | 32% | 40% |

| | | |
|---|---|---|
| • Production US | 72 | 64 Quad BTU |
| • Consumption US | 102 | 76 Quad. BTU |